University of Kerala
First Degree Programme in Mathematics
Model Question Paper
Semester I
MM 1141: Methods of Mathematics
(2023 Admission onwards)

Time: 3 Hours Max. Marks: 80

Section - I

*All **ten** questions are compulsory. Each question carries **1** mark.*

1. Whenever we say f is increasing on an interval?
2. If $f''(a)$ exists and f has an inflection point at $x = a$, then $f''(a) = \ldots$.
3. Assume that f is differentiable everywhere. Determine whether the statement is true or false. If f is decreasing on $[0, 2]$, then $f(0) > f(1) > f(2)$.
4. If $f''(x_0) = 0$ and $f''(x_0) > 0$, then f has a ... at x_0
5. If f has an absolute extremum on an open interval (a, b), then it must occur at a ... point of f.
6. What is meant by a *statement*?
7. Write the antecedent and the consequent of the following sentence: *If it stops raining by Saturday, then I will go to the football game.*
8. Write the well ordering principle.
9. State true or false: There are infinitely many primes.
10. If $(a, b) = 1 = (a, c)$, then $(a, bc) = \ldots$.

Section - II

*Answer **any eight** questions. Each question carries **2** marks.*

11. Suppose that $f(x)$ has derivative $f'(x) = (x-4)^2 e^{-x/2}$. Then $f''(x) = -\frac{1}{2}(x-4)(x-8)e^{-x/2}$. The function f is increasing on the interval(s) ....

12. Let $y = \sqrt{x}$. Find formulas for Δy and dy.

13. Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.

14. Let $s(t) = t^3 - 6t^2$ be the position function of a particle moving along an s-axis, where s is in meters and t is in seconds. Find the acceleration function $a(t)$.

15. State Rolle's Theorem.

16. Find all critical points of $f(x) = x^3 - 3x + 1$.

17. Whenever we say a function f is concave up? concave down?

18. Write an example of a biconditional statement.

19. Construct a truth table for the compound statement

$$\sim (p \wedge q) \Leftrightarrow [(\sim p) \vee (\sim q)].$$

20. Write division algorithm for integers.

21. Write six consecutive integers that are composites.

22. Find the canonical decomposition of 2520.

Section - III

*Answer **any six** questions. Each question carries **4** marks.*

23. Find all relative extrema of $f(x) = 3x^{5/3} - 15x^{2/3}$.

24. Determine by inspection whether $p(x) = 3x^4 + 4x^3$ has any absolute extrema. If so, find them and state where they occur.

25. Show that the function $f(x) = \frac{1}{4}x^3 + 1$ satisfies the hypotheses of the Mean-Value Theorem over the interval $[0, 2]$ and find all values of c in the interval $(0, 2)$ at which the tangent line to the graph of f is parallel to the secant line joining the points $(0, f(0))$ and $(2, f(2))$.

26. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?

27. Prove that any postage of $n(\geq 2)$ cents can be made with two- and three-cent stamps.

28. Find the number of positive integers ≤ 3000 and divisible by 3, 5, or 7.

29. Show that every integer $n \geq 2$ has a prime factor.

30. Write the negation of each of the following

    (a) There exists a positive number x such that $x^2 = 5$.

    (b) For every positive number M, there is a positive number N such that $N < 1/M$.

    (c) If $n \geq N$, then $|f_n(x) - f(x)| \leq 3$ for all x in A.

    (d) No positive number x satisfies the equation $f(x) = 5$.

31. (a) Use a truth table to verify that $p \Rightarrow q$ and $\sim q \Rightarrow \sim p$ are logically equivalent.

    (b) Is $p \Rightarrow q$ logically equivalent to $q \Rightarrow p$?

Section - IV

*Answer **any two** questions. Each question carries **15** marks.*

32. (a) The diameter of a polyurethane sphere is measured with percentage error within $\pm 0.4\%$. Estimate the percentage error in the calculated volume of the sphere.

    (b) Use the first and second derivatives of $f(x) = x^3 - 3x^2 + 1$ to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Locate all inflection points.

33. (a) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

    (b) You are driving on a straight highway on which the speed limit is 55 mi/h. At 8:05 a.m. a police car clocks your velocity at 50 mi/h and at 8:10 a.m. a second police car posted 5 mi down the road clocks your velocity at 55 mi/h. Explain why the police have a right to charge you with a speeding violation.

34. (a) Let a be any integer and b a positive integer. Show that there exist unique integers q and r such that $a = b{\cdot}q + r$ where $0 \leq r < b$.

    (b) Using Euclidean algorithm evaluate $(4076, 1024)$.

35. Explain any three proof techniques with one example for each.