Board of Studies in Mathematics (UG)
UNIVERSITY OF KERALA

First Degree Programme in
MATHEMATICS
under Choice Based Credit and Semester System

REVISED SYLLABUS
2014 admission

Structure of Core Courses

Sem	Course Code	Course title	Instr.hrs. per week	Credit
I	MM 1141	Methods of Mathematics	4	4
II	MM 1221	Foundations of Mathematics	4	3
III	MM 1341	Algebra and Calculus-I	5	4
IV	MM 1441	Algebra and Calculus-II	5	4
V	MM 1541	Real Analysis-I	5	4
	MM 1542	Complex Analysis I	4	3
	MM 1543	Differential Equations	3	3
	MM 1544	Vector Analysis	3	3
	MM 1545	Abstract Algebra I	5	4
	MM 1551	Open Course	3	2
		Project	2	
VI	MM 1641	Real Analysis-II	5	4
	MM 1642	Linear Algebra	4	3
	MM 1643	Complex Analysis II	3	3
	MM 1644	Abstract Algebra II	3	3
	MM 1645	Computer Programming (Pract.)	5	4
	MM 1651	Elective Course	3	2
	MM 1646	Project	2	4

Structure of Open Courses

Sem	Course Code	Course title	Instr.hrs. per week	Credit
V	MM 1551.1	Operations Research	3	2
V	MM 1551.2	Business Mathematics	3	2
V	MM 1551.3	Actuarial Science	3	2

Structure of Elective Courses

Sem	Course Code	Course title	Instr.hrs. per week	Credit
VI	MM 1661.1	Graph Theory	3	2
VI	MM 1661.2	Fuzzy Mathematics	3	2
VI	MM 1661.3	Mechanics	3	2

Structure of the Complementary Courses

Complementary Course in Mathematics for First Degree Programme in Physics

Course Code	Sem.	Title of Course	Contact hrs/week	No. of Credits
MM 1131.1	1	Differentiation and Analytic Geometry	4	3
MM 1231.1	2	Integration and Vectors	4	3
MM 1331.1	3	Theory of Eqs., Differential Eqs., and Theory of Matrices	5	4
MM 1431.1	4	Complex Analysis, Fourier Series and Transforms	5	4

Complementary Course in Mathematics for First Degree Programme in Chemistry

Course Code	Sem.	Title of Course	Contact hrs/week	No. of Credits
MM 1131.2	1	Differentiation and Matrices	4	3
MM 1231.2	2	Integration, Differential Eqs. and Analytic Geometry	4	3
MM 1331.2	3	Theory of Eqs.and Vector Analysis	5	4
MM 1431.2	4	Abstract Algebra and Linear Transformations	5	4

Complementary Course in Mathematics for First Degree Programme in Geology

Course Code	Sem.	Title of Course	Contact hrs/week	No. of Credits
MM 1131.3	1	Differentiation and Theory of Equations	4	3
MM 1231.3	2	Integration, Differential Eqs. and Matrices	4	3
MM 1331.3	3	Analytic Geometry, Complex Nos. and Abstract Algebra	5	4
MM 1431.3	4	Vector Analysis and Fourier Series	5	4

Complementary Course in Mathematics for First Degree Programme in Statistics

Course Code	Sem.	Title of Course	Contact hrs/week	No. of Credits
MM 1131.4	1	Theory of Eqs., Infinite Series and Analytic Geometry	4	3
MM 1231.4	2	Differential Calculus	4	3
MM 1331.4	3	Integration and Complex Nos.	5	4
MM 1431.4	4	Linear Algebra	5	4

Complementary Course in Mathematics for First Degree Programme in Economics

Course Code	Sem.	Title of Course	Contact hrs/week	No. of Credits
MM 1131.5	1	Mathematics for Economics I	3	2
MM 1231.5	2	Mathematics for Economics II	3	3
MM 1331.5	3	Mathematics for Economics III	3	3
MM 1431.5	4	Mathematics for Economics IV	3	3

Syllabus for the First Degree Programme in Mathematics
of the University of Kerala

Semester I
Methods of Mathematics

CODE: MM 1141 — Instructional hours per week: 4

No.of credits: 4

Module I Algebra

Text : Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*, Second Edition, Springer

In this part of the course, we study the basic properties of natural numbers, traditionally called *Theory of Numbers*. It is based on Chapters 2–5 of the text. Students should be encouraged to read the textbook and try to do the problems on their own, to gain practice in writing algebraic proofs. All the problems and exercises at the end of each section are to be discussed.

We start with the methods of proofs by induction, as in Sections A and B of Chapter 2. The intuitive idea that these methods give a scheme of extending a result from one natural number to the next *independent of the number under consideration* should be stressed. The fact that the second principle is easier in some cases should be illustrated through examples such as Example 1 of Section B. *The logical equivalence of these two methods (Theorems 1 and 2 of Section B) need not be discussed.*

We then pass onto THE WELL ORDERING PRINCIPLE, as in Section C. Example E1, Theorem 1 and Proposition 3 should be discussed with proofs based on this principle. *The deduction of this principle from the principle of induction, as in Theorem 2, need not be done.* Thus the two principles of induction and the well-ordering principle need only be discussed as intuitively obvious properties of natural numbers.

Before introducing the DIVISION THEOREM, as in Section D, the usual process of long division to get the *quotient* and *remainder* must be recalled through examples and the formal proof of this theorem should be linked to these examples. After proving the this theorem and the UNIQUENESS PROPOSITION as in this section, the representation of natural numbers in different bases can be explained as in Section E. *The last section of Chapter 2 on operations in different bases (Section F) need not be discussed.*

The idea of the GREATEST COMMON DIVISOR of two natural numbers, studied in elementary class, is to be recalled next and the *existence* of a such a number justified, as in Section A of Chapter 3. The idea of *coprimality* is also to be considered here. Some of the important properties of coprime numbers, as in Exercises E9, E10 and E11 must be discussed. Next, EUCLID'S ALGORITHM and some of its applications are to be discussed, as in Section B. After discussing the theoretical consequences of Euclid's Algorithm, namely *Bezout's Identity* and its corollaries, as in Section C, its practical use in solving indeterminate equations of the first degree is to be discussed, as in the text. (See also `http://en.wikipedia.org/wiki/Diophantine_equation`) *The last two sections of this chapter on the efficiency of Euclid's Algorithm (Section D) and on incommensurability (Section E) need not be discussed.*

A discussion on primes and THE FUNDAMENTAL THEOREM ON ARITHMETIC, as given in the first three section of Chapter 4 are to be done next. *The last section of this chapter on primes in an interval need not be discussed.*

Finally we introduce the new idea of congruences as in Chapter 5. The fact that when an integer is divided by another, the dividend is congruent to the remainder modulo the divisor should be emphasized. In discussing the basic properties of congruences the fact that the cancellation of common factors does not hold in general for congruences should be emphasized and illustrated through examples. This part of the course is based on Sections A, B, C of Chapter 5.

Module 2 Calculus

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

In this part of the course, the basic ideas of differentiation of real valued functions are considered. It is based on Chapters 1–3 of the text.

We start with the intuitive idea of a function as the dependence of one quantity on another as in the subsection titled FUNCTIONS of Section 1.1 of the text and pass on to Definitions 1.1.1 and 1.1.2. We next discuss basic properties of functions, as in Section 1.2. It must be emphasized through illustrations that not all equations connecting two variables give one variable as a function of the other, as in Example 1 of Section 1.2 of the text. (The notion of *explicit* and *implicit* definitions of functions and their graphs, as given in the first two parts of Section 3.6 can be discussed here itself.) Functions defined piecewise and their graphs must be specially mentioned and illustrated. Approximate solutions to problems through graphical methods are to be explained as in Example 7 of the section. Section 1.3 on using computers may be skipped, but the use of computers in plotting graphs should be demonstrated, using Open Source Software such as GeoGebra or Gnuplot.

(See also `http://www-groups.dcs.st-and.ac.uk/~history/Curves/Curves.html`)

Some of the ideas in Section 1.4, such as arithmetic operations on functions, maybe familiar to the students, but they should be reviewed. Other ideas such as symmetry, stretching and compression and translation maybe new and should be emphasized. Section 1.5 named LINES maybe supplemented with Appendix C, COORDINATE PLANES AND LINES. *Section 1.6 on families of functions and Section 1.7 on mathematical modelling need not be discussed.* But parametric equations, especially that of the cycloid, must be discussed in detail, as in Section 1.8.

Limits and continuity are concepts introduced in Higher Secondary class. In this course, the intuitive description of of these ideas are to be reinforced through tabulation and plotting and illustrated through examples, as in Sections 2.1–2.3. *The rigorous description of limits, as in Section 2.4, need not be discussed.* Sections 2.5 and 2.6 on continuity must be discussed.

The notion of differentiation is also familiar to the students. Here, this idea is to be re-introduced through applications as in the first two sections of Chapter 3.

(See also `http://en.wikipedia.org/wiki/History_of_calculus`) The discussion of VELOCITY AND SLOPES at the beginning of Section 3.1 maybe based on Example 1 of Section 2.1, instead of the unfamiliar bell-pulling example. Much of the material in Sections 3.3–3.7 maybe already seen, but they should be reviewed, emphasizing the graphical meaning and applications. The idea of implicit differentiation should be made clear, as in Section 3.6. *The last section on approximations, Section 3.8, need not be discussed.*

Module 3 Analytic Geometry

Text: Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

This part of the course is a detailed discussion on conics, based on Sections 11.4, 11.5 and parts of 11.6 of the text. Students are introduced to the standard equation of the conics in the Higher Secondary class, but little else on conics. Here we start with the geometrically unified description of conics as sections of a cone, as in the subsection conic sections of Section 11.4 of the text (see also `http://en.wikipedia.org/wiki/Conic_sections`) and pass on to the description subsection DEFINITION OF THE CONIC SECTIONS. Various problems in EXERCISE SET 11.4 on practical applications of conics should be discussed. Theorem 11.6.1 and the discussions following it are to be discussed next. (The connection between the description of conics as sections of cone and using the focus-directrix property can be in `http://en.wikipedia.org/wiki/Dandelin_spheres`) Finally, the geometric and algebraic description of conics tilted with respect to the coordinate axes are discussed as in Section 11.5, culminating in Theorem 11.5.2 characterizing the graphs of all second degree equations in two variables.

The final aim of this part is to give a complete characterization of graphs of second degree equations in two variables as given in Theorem 11.5.2, thus giving an algebraically unified description of conics.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.

2. Thomas and Finney,*Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

3. S.Lang, *A first Calculus*, Springer.

Distribution of instructional hours:

Module 1: 24 hours; Module 2: 36 hours; Module 3: 12 hours

Semester II

Foundations of Mathematics

CODE: MM 1221 Instructional hours per week: 4

No.of credits: 3

Module I Algebra

Text : Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*, Second Edition, Springer

We continue the study of the theory of numbers, based on parts of Chapters 5–7 and Chapters 9–10 of the text. (Chapter 8, discussing abstract ideas is postponed to the next semester.)

We start with Sections D and E of Chapter 5, which discuss more properties and applications of the idea of congruence introduced in the first semester course. We then pass on to the idea of congruence classes and related ideas, as in Chapter 6 of the text. The notion of Congruence modulo m, done in the first semester, is now introduced as an equivalence relation and the congruence classes modulo m are discussed through examples such as $\mathbb{Z}/2\mathbb{Z}$ and $\mathbb{Z}/12\mathbb{Z}Z$ (clock arithmetic), leading to the general set $\mathbb{Z}/m\mathbb{Z}$. Here we can recall the ideas of equivalence relation (learnt in Higher Secondary class) and partition and the relation between the two. The sections named RATIONAL NUMBERS, EQUIVALENCE CLASSES and NATURAL NUMBERS of Chapter 1 should be used to supplement this discussion. As applications, only Section A of Chapter 7 on ROUND ROBIN TOURNAMENTS and Section C on TRIAL DIVISION need be discussed.

Next we move on to Fermat's and Euler's Theorems, as in Chapter 9. Only the first four sections of this chapter need be done. (The other sections are to be discussed in the next semester.) In Section C, exercises E7–E10 on the computation of Euler's phi function must be done and used to compute the phi-value of some specific numbers see also Bernard and Child, *Higher Algebra*. As an applications, only FINDING HIGHER POWERS MODULO m (Section D of Chapter 9, see also `http://en.wikipedia.org/wiki/RSA`), RSA CODES Mersenne Numbers and Fermat Numbers (Section C of Chapter 10) need be done.

Module 2 Calculus

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

In this part, we continue the discussion on calculus and analytic geometry started in the first semester. It is based on parts of Chapters 4–8 and Chapter 11 of the text.

We start with the discussion on how the derivative of a function can be used to visualize the graph of the function in better detail, as described in Sections 4.1–4.3 of the text. We then discuss how the ideas of maxima and minima can be used to solve practical problems, as in Section 4.5. *Sections 4.4, 4.7 and 4.8 need not be discussed.*

We next introduce the idea of integration as anti-differentiation, as in Definition 5.2.1. As motivation for this idea, the first two subsections, FINDING POSITION AND VELOCITY BY INTEGRATION and UNIFORMLY ACCELERATED MOTION of Section 5.7 can be used. *The lat two subsections of Section 5.2,* INTEGRATION FROM THE VIEWPOINT OF DIFFERENTIAL EQUATIONS *and* DIRECTION FIELDS, *need not be discussed.* After completing Sections 5.2 and 5.3, we turn to the area problem, as in Section 5.1. We pass on to the subsections DEFINITION OF AREA and NET SIGNED AREA of Section 5.4. Only Definitions 5.4.3 and 5.4.5 of this section and the discussions preceding these need be discussed. We then discuss the subsection RIEMANN SUMS AND THE DEFINITE INTEGRAL of Section 5.5. Only Definition 5.5.1 and Theorems 5.5.4 and 5.5.5 of this section need be discussed. The connection between anti-differentiation and Riemann integration is to be discussed next, as in the subsection THE FUNDAMENTAL THEOREM OF CALCULUS of Section 5.6. *The proof of Theorem 5.6.1 and the remaining parts of this section need not be discussed.* But Sections 5.7 and 5.8 are to be discussed in full. Applications of integration comes next, as in Sections 6.1–6.5 of the test. *Sections 6.6 and 6.7 need not be discussed.*

Though the idea of inverse functions is introduced in the Higher Secondary class, this has to be done in a more thorough manner as in Section 7.1. Also, the ideas have to be graphically interpreted. Before discussing the exponential and logarithmic functions, the idea of irrational exponents has to be made clear, as in Section 7.2. After Section 7.3 on differentiation and integration of the exponential and logarithmic functions, Section 7.6 on inverse trigonometric functions, Section 7.7 on LH́ospital's Rule and Section 7.8 on hyperbolic functions are to be done in full. *Sections 7.4 and 7.5 need not be discussed.*

Various techniques of integration are to be considered next, as in Sections 8.1–8.5. Then improper integrals are to be discussed as in Section 8.8. *The other sections, 8.6 and 8.7 need not be discussed.*

Module 3 Analytical Geometry

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

In this part of the course, we introduce polar coordinates as in Section 11.1 of the text. Areas in polar coordinates are to be done as in Section 11.3 and the polar equations of conics as in Section 11.6. The subsection APPLICATIONS IN ASTRONOMY must also be discussed.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney,*Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. S.Lang, *A first Calculus*, Springer.

Distribution of instructional hours:

Module 1: 24 hours; Module 2: 36 hours; Module 3: 12 hours

Semester III

Algebra and Calculus I

CODE: MM 1341 Instructional hours per week: 5

No.of credits: 4

Module I Algebra

Text : Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*, Second Edition, Springer

Continuing the discussion on number theory in the first two semesters, here we make first contact with the part of mathematics currently called *Abstract Algebra*. It is based on parts of Chapters 8, 9, and 12 of the text.

Contrary to the usual stand-alone courses on abstract algebra, we introduce rings before groups, since the former arise naturally as generalizations of number systems. Sections A and B of Chapter 8, (including the problems) are to be discussed in full. *In section C, the definition of characteristic and the rest of the portions need not be discussed.* More examples of rings and exercises on homomorphism can be given to get a clear idea of the concepts.

Next comes a discussion on the units of the ring of congruence classes leading to the definition of an abstract group and then the GROUP OF UNITS of an abstract ring, as in Section E and Section F of Chapter 9. This culminates in the ABSTRACT FERMAT'S THEOREM, as in Section E. *The proofs of generalized associativity or generalized commutativity need not be discussed.* But the fact that a set G with an associative multiplication is a group, if it either has the identity and inverse properties or has the cancellation and solvability properties has to be proved (see T. W .Hungerford, *Algebra*). The exponent of an Abelian group, as in Section 9F also has to be discussed. As an illustration of the interplay between number theory and abstract algebra, we consider the THE CHINESE REMAINDER THEOREM, as in Section A of Chapter 12. Only the first part and the problems E1, E2, E3 and E4 of this section need be discussed , *The alternate method of reducing all the congruences to one need not be considered.* As another application, the multiplicative property of the phi function discussed earlier must be redone (Corollary 3 of Section C). The square roots of 1 modulo some integer, as in Section C of Chapter 12 must also be discussed.

REFERENCES:

1. J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications
2. I N Herstein, *Topics in Algebra*, Vikas Publications
3. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications
4. D A R Wallace, *Groups, Rings and Fields*, Springer
5. Jones and Jones, *Number Theory*, Springer

Module 2 Analytic Geometry

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

In this part of the course, we consider equations of surfaces and curves in three dimensions. It is based on Chapter 12 of the text.

Students have had an introduction to analytic geometry in three dimensions, such as the equations to planes and lines, and to vectors in their Higher Secondary Classes. These must be reviewed with more illustrations. Here the aid of a plotting software becomes essential. The Free Software GNUPLOT mentioned earlier has such 3D capabilities. (see also `http://mathworld.wolfram.com/topics/Surfaces.html`)

After discussing SPHERES and CYLINDRICAL SURFACES as in Section 12.1, We pass on to a discussion of VECTORS, as in Section 12.2. The physical origins of the concept must be emphasized as in the subsection, VECTORS IN PHYSICS AND ENGINEERING. The definition of vector addition can be motivated by the discussion given in the subsection, RESULTANT OF CONCURRENT FORCES which may be familiar to students from their high school physics. All the sections of the chapter are to be discussed in the same spirit, emphasizing both the physical and geometrical interpretations.

Module 3 Calculus

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Here we extend the operations of differentiation and integration to *vector valued functions* of a real variable, based on Chapter 13 of the text.

All sections of this Chapter must be discussed, with emphais on geometry and physics, as in the text. The problems given in various exercise sets should be an essential part of the course. Exercises 17 (a) and 17 (b) of Exercise Set 13.5 on curvature of plane curves and some of its applications in the subsequent exercises must be discussed in detail.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. S.Lang, *A first Calculus*, Springer.

Distribution of instructional hours:

Module 1: 36 hours; Module 2: 27 hours; Module 3: 27 hours

Semester IV

Algebra and Calculus II

CODE: MM 1441 Instructional hours per week: 5

No.of credits: 4

Module I Algebra

Text : Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*, Second Edition, Springer

Continuing the study of rings in the last semester, here we introduce polynomials as another example. This part of the course is based on Chapters 14, 15 and parts of Chapter 16 of the text.

After reviewing the idea of polynomials studied in High School, we introduce polynomials over a commutative ring. The distinction between polynomial as an algebraic expression and polynomial as a function should be emphasized, as in the section POLYNOMIALS AND FUNCTIONS of Chapter 14. All sections of Chapters 14 and 15 are to be discussed.

We then briefly consider irreducible polynomials with real coefficients. After discussing the dependence of irreducibility on the field of coefficients as in the beginning of Chapter 16, we pass on to Section C. The reducibility of polynomials of degree greater than 2 over real numbers must be mentioned, but *Euler's proof for degree 4 need not be discussed*. The fact that the root of a polynomial gives a factor leads to the consideration of roots as in Section E. (*Complex numbers, as in Section D need not be discussed here.*) The origin of complex numbers in the study of cubic equations must be emphasized. (See also, Paul J Nahin, *An Imaginary Tale: The Story of* $\sqrt{-1}$) The unsolvability of higher degree polynomials by radicals, mentioned at the end of this section, must be noted. The FUNDAMENTAL THEOREM OF ALGEBRA must next be discussed. *This theorem need not be proved*, but Euler's real version (COROLLARY 1) must be proved based on this, as in the text.

REFERENCES:

1. J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications

2. I N Herstein, *Topics in Algebra*, Vikas Publications

3. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications

4. D A R Wallace, *Groups, Rings and Fields*, Springer

5. Jones and Jones, *Number Theory*, Springer

Module 2 Calculus

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

In this part of the course, we consider the calculus of functions of two variables. It is based on Chapter 14 and Chapter 15 of the text. The geometric interpretation of the ideas should be emphasized throughout, with the aid of plotting software such as Gnuplot.

After a discussion of functions of two variable and their graphs, as in the first section of Chapter 14, we discuss the concepts of limit and continuity of such functions. We then move on to a discussion of differentiation of functions of two variables, as in Sections 14.1–14.3, 14.5 and 14.8–9. *Section 14.4 on differentiability and differentials and Section 14.6 on directional derivatives and Section 14.7 on tangent planes need not be discussed.*

Integration in space is to be done as in Sections 1–5 of Chapter 15. *The last three sections of Chapter 15 need not be discussed.*

References:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.

2. Thomas and Finney,*Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

3. S.Lang, *A first Calculus*, Springer.

Distribution of instructional hours:

Module 1: 36 hours; Module 2: 54 hours

Semester V

Real Analysis I

CODE: MM 1541 Instructional hours per week: 5

No.of credits: 4

Text : R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, Third Edition, Wiley

In this course, we discuss the notion of real numbers and the ides of limits in a formal manner. Many of the topics discussed in this course were introduced somewhat informally in earlier courses, but in this course, the emphasis is on mathematical rigor. It is based on Chapters 2–4 of the text.

In teaching this course, all ideas should be first motivated by geometrical considerations and then deduced algebraically from the axioms of real numbers as a complete ordered field. Also, the historical evolution of ideas, both in terms of physical necessity and mathematical unity should be discussed. Thus the course emphasizes the dialectic between practical utility and logical rigor in general, and within mathematics, that between geometric intuition and algebraic formalism.

Throughout the course, examples and exercises in the text should be used to illustrate the ideas discussed. Students should be encouraged to do problems on their own, to gain practice in writing rigorous proofs.

Module 1

The first step is to make precise the very concept of number and the rules for manipulating numbers. The course can start with a historical overview of how different kinds of numbers were constructed in different periods in history, depending on the physical or mathematical needs of the age. (See for example, the three articles on real numbers at `www-groups.dcs.st-and.ac.uk/~history/Indexes/Analysis.html`) A discussion on how real numbers are conceived as lengths and hence as points on a line should follow this. The efforts to approximate irrational numbers by rational numbers, in the familiar instances such as $\sqrt{2}$ and π can lead to the modern decimal representation and this gives semi-rigorous definitions of operations on real numbers.

The realization of the set $\mathbb{R}$ of real numbers as a field can be introduced at this stage and compared with the set $\mathbb{Q}$ of rational numbers, as in 2.1.1–2.1.4 of the textbook. The idea of order in $\mathbb{Q}$ and $\mathbb{R}$ must be introduced next, as in 2.1.5–2.1.13 of the textbook. The notion of absolute value and that of a neighborhood, as in 2.2.1–2.2.9 of the textbook comes next.

The discussion of the COMPLETENESS PROPERTY OF $\mathbb{R}$ requires some care. The version given in 2.3.6 of the text is highly counter-intuitive as an axiom. Instead, Instead the following version due to Dedekind can be used:

If the set of real numbers is split into two non-empty sets such that every number in one set is less than every number in the other, then either the first set contains a least number or the second set contains a largest number

And this can be easily interpreted geometrically as a line considered as a set of points. (See R. Dedekind, *Essays on The Theory of Numbers*, available as a freely downloadable e-book at `http://www.gutenberg.org/etext/21016`) The SUPREMUM PROPERTY of $\mathbb{R}$ can easily *proved* as a consequence of this axiom.

It should be emphasized at this point that in this course, the only assumptions we make about $\mathbb{R}$ are the axioms of a complete ordered field and every definition we make would be given in terms of these and every result we propose would be deduced from these axioms (and the theorems proved earlier). The remaining part of Section 2.3 and Section 2.4 in full are to be discussed as in the test. *In Section 2.5, the subsections,* THE UNCOUNTABILITY OF $\mathbb{R}$, BINARY REPRESENTATIONS, DECIMAL REPRESENTATIONS, PERIODIC DECIMALS *and* CANTOR'S SECOND PROOF *need not be discussed*.

Module 2

We then pass on to the idea of limits of sequences and series, as in Chapter 3 of the text. It should be supplemented by Sections 10.2 and 10.4 of the calculus text by Anton (used in earlier semesters) to provide motivation, illustrative examples and more exercises.

Module 3

Limits of functions are to be discussed as in Chapter 4 of the text. Before introducing the rigorous definition of limits, the informal description of these ideas through graphs, as done in the earlier calculus courses should be recalled. Also, the various theorems should be illustrated through examples and exercises given in the text. Plotting software such as Geogebra can be used to plot the various functions discussed in Chapter 4.

References

1. A. D. ALEXANDROV et al., *Mathematics:Its Content, Methods and Meaning*, Dover
2. R. DEDEKIND, *Essays on The Theory of Numbers*, available as a freely dowloadable e-book at `http://www.gutenberg.org/etext/21016`)
3. W. RUDIN, *Principles of Mathematical Analysis*, Second Edition, McGraw-Hill
4. A. E. TAYLOR, *General Theory of Functions and Integration*, Dover

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

Semester V

Complex Analysis I

CODE: MM 1542 — Instructional hours per week: 4

No.of credits: 3

Text : Joseph Bak and Donald J. Newman, *Complex Analysis*. Third Edition, Springer

In this course, we discuss the basic properties of complex numbers and extend the notions of differentiation and integration to complex functions. It is based on Chapters 1–4 of the text. Examples and exercises in the text forms an integral part of the course.

Module 1

The basic operations on complex numbers are familiar to the students from their Higher secondary course. Also, the historical motivation for complex numbers is briefly touched upon in MODULE 1 of the fourth-semester course ALGEBRA AND CALCULUS IV. So, the present course can start with a brief review of the INTRODUCTION and a discussion on the representation of complex numbers as ordered pairs of real numbers as in Section 1.1. The other sections of this chapters are to be discussed in order. *The definition of uniform convergence and* 1.9 M-TEST *in Section 1.4 need not be discussed. Also,* STEREOGRAPHIC PROJECTION *as in Section 1.5 need not be discussed*, but infinite limits should be introduced (I.11 DEFINITION). The use of complex numbers in number theory and geometry are to be illustrated using Exercises 9, 10 and 14 of this chapter.

We then pass on to the definition of complex functions, starting with polynomials as Chapter 2..The difference between a polynomial function of *two real variables* and that of *a single complex variable* should be emphasized as in the INTRODUCTION to this chapter. Also, in discussing ANOTHER WAY OF RECOGNIZING ANALYTIC POLYNOMIALS preceding 2.2 Definition, it should be noted that the field operations allow us only to define upto *rational functions* of complex numbers and that expressions like $\cos(x + iy)$ are meaningless at this stage. In discussing POWER SERIES as in Section 2.8, *the proof of* 2.8 THEOREM *and the comment following the proof about uniform convergence need not be discussed.* Examples 1–3 following this are to be emphasized as signifying the behaviour of different power series *on* the circle of convergence. The remainig part of Chapter 2 should be discussed in full.

Module 2

In Chapter 3 on ANALYTIC FUNCTIONS, *the proof of 3.2 Proposition on the sufficiency of Cauchy-Riemann Equations for analyticity need not be done.* Except for this, Chapter 3 must be done in full. Exercises 21–23 on the power series expansions of the exponential function and the sine and cosine functions must also be discussed in detail.

Module 3

In Chapter 4, the definition of the integral of f along C (4.3 DEFINITION of the text) should be motivated as limit of the Riemann sums of the form $\sum f(z_k)(z_k - z_{k-1})$ (see for example, the MIT OPENCOURSEWARE video of LECTURE 5 of PART I CALCULUS under CALCULUS REVISITED). *In Section 4.1, the result on the integral of uniform limit* (4.11 PROPOSITION) *need not be discussed*. Section 4.2 is to be discussed in full.

References

1. JAMES BROWN AND RUEL CHURCHILL, *Complex Variables and Applications*, Eighth Edition, McGraw-Hill

2. J. M. HOWIE, *Complex Analysis* Springer

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

Semester V

Differential Equations

CODE: MM 1543 Instructional hours per week: 3

No.of credits: 3

Texts : 1. Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

2. Erwin Kreyszig, *Advanced Engineering Mathematics*, Eigth Edition, Wiley-India

In this course, we discuss how differential equations arise in various physical problems and consider some methods to solve first order differential equations and second order linear equations. It is based on parts of Chapters 5 and 9 of [1] and Chapter 2 of [2].

Module 1

In this module we discuss first order equations and is based on [1]. We start with some simple examples of physical situatons in which differential equations arise, using some of the examples of Section 9.3. This is to be followed by the last two subsections of Section 5.2, INTEGRATION FROM THE VIEWPOINT OF DIFFERENTIAL EQUATIONS and DIRECTION FIELDS includeing problems related to these ideas from EXERCISE SET 5.2. We next consider first order differential equations as in Sections 9.1–9.3. Then we discuss EXACT DIFFERENTIAL EQUATIONS as in Section 1.5 of [2].

Module 2

Second order linear differential equations are discussed in this module and it is based on Chapter 2 of [2]. More precisely, Sections 2.1–2.3 and Sections 2.4–2.11 must be done in detail, including relevant problems. *Section 2.3 on* DIFFERENTIAL OPERATORS *need not be discussed*

References

1. G. F. SIMMONS, *Differential Equations with applications and Hystorical notes*, Tata McGraw-Hill, 2003

2. PETER V. O' NEIL, *Advanced Engineering Mathematics*, Thompson Publications, 2007

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours

Semester V

Vector Analysis

Code: MM 1544 — Instructional hours per week: 3

No.of credits: 3

Text : Howard Anton, et al, *Calculus*, 7^{th} Edn, John Wiley

In this course, we consider some advanced parts of vector calculus. It is based on parts of Chapter 14 and Chapter 16 of the text. The physical motivation and interpretation of the various mathematical concepts should be emphasized throughout, as in the text.

Module 1

We begin with the notion of directional derivatives as in Section 14.6. *The last subsection on derivative of a function of several variables need not be discussed*. We then pass on to the definition of a vector field and its divergence and curl, as in Section 16.1. The del and Laplacian operators must also be discussed. We next discuss line integrals, as in Section 16.2 and then conservative vector fields, as in Section 16.3. This module of the course ends with a discussion of Green's Theorem, as in Section 16.4.

Module 2

In this module, we introduce the notion of a surface integral and discuss Gauss's Theorem and Stoke's Theorem and their applications, as in Sections 16.5–16.8 of the text

References:

1. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
2. Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition, John Wiley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007
4. Michael D. Greenberg, *Advanced Engineering Mathematics*, PearsonEducation, 2002.

Distribution of instructional hours:

Module 1: 27 hours; Module 2: 27 hours

Semester V

Abstract Algebra I

CODE: MM 1545 Instructional hours per week: 5

No.of credits: 4

Text : John B. Fraleigh, *A First Course in Abstract Algebra*. Seventh Edition, Narosa

Students introduced to some elements of Abstract Algebra in Semester IV are now ready to do it rigorously. In this course, we discuss the basics of abstract group theory, based on Sections 2–10 of the text.

Students should be given training to write proofs and to do problems, based on axioms. The recommended text contains lots of examples and exercises. Most of the problems in this text are computational and hence the student can try them as assignments with proper guidance of the teacher.

Module 1

The course begins with section 0, which can be reviewed quickly. *The subsection on* CARDINALITY *need not be discussed.* We then move on to Section 2 on binary relations (*skipping Section 1.* The ideas of *binary operation on a set*, *well definedness of a binary operation* and *a set closed under a binary operation* should be emphasized. Isomorphisms of binary structures should be done in detail, as in Section 3. After recalling the idea of abstract groups introduced in the previous semester, Section 4 on groups, Section 5 on subgroups and Section 6 on cyclic groups must be done in full. *Section 7,* GENERATING SETS AND CAYLEY DIGRAPHS, *need not be discussed.*

Module 2

We next consider the group of permutations in detail, as in Section 8–10 (Chapter II) and cosets and Lagrange's Theorem, as in Section 10. The first part of Section 11 on direct products of groups should also be discussed. *The second part,* FINITELY GENERATED ABELIAN GROUPS *and the entire Section 12,* PLANE ISOMETRIES *need not be discussed.*

REFERENCES:

1. I N Herstein, *Topics in Algebra*, Vikas Publications

2. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications

3. D A R Wallace, *Groups, Rings and Fields*, Springer

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 45 hours; Module 2: 45 hours

Semester V

Operations Research (Open Course)

CODE: MM 1551.1

Instructional hours per week: 3
No. of Credits: 2

Module 1 LINEAR PROGRAMMING: Formulation of Linear Programming models, Graphical solution of Linear Programs in two variables, Linear Programs in standard form - basic variable - basic solution- basic feasible solution -feasible solution, Solution of a Linear Programming problem using simplex method (Since Big-M method is not included in the syllabus, avoid questions in simplex method with constraints of $\geq$ or $=$ type.)

Module 2 TRANSPORTATION PROBLEMS: Linear programming formulation - Initial basic feasible solution (Vogel'sapproximation method/North-west corner rule) - degeneracy in basic feasible solution - Modified distribution method - optimalitytest.

ASSIGNMENT PROBLEMS: Standard assignment problems - Hungarian method for solving an assignment problem.

Module 3 PROJECT MANAGEMENT: Activity -dummy activity - event - project network, CPM (solution by network analysis only), PERT.

TEXT: Ravindran - Philps - Solberg: Operations Research- Principles and Practice

REFERENCE:

Hamdy A Taha: *Operations Research*

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours

Semester V

Business Mathematics (Open Course)

CODE: MM 1551.2 — Instructional hours per week: 3
No. of Credits: 2

Module 1 Basic Mathematics of Finance: Nominal rate of Interest and effective rate of interest, Continuous Compounding, force of interest, compound interest calculations at varying rate of interest, present value, interest and discount, Nominal rate of discount, effective rate of discount, force of discount, Depreciation.

(Chapter 8 of Unit I of text- Sections: 8.1, 8.2, 8.3, 8.4. 8.5, 8.6, 8.7, 8.9)

Module 2 Differentiation and their applications to Business and Economics: Meaning of derivatives, rules of differentiation, standard results (basics only for doing problems of chapter 5 of Unit 1) (Chapter 4 of unit I of text- Sections: 4.3, 4.4, 4.5, 4.6)

Maxima and Minima, concavity, convexity and points of inflection, elasticity of demand, Price elasticity of demand (Chapter 5 of Unit I of text - Sections: 5.1, 5.2, 5.3, 5.4, 5.5. 5.6, 5.7)

Integration and their applications to Business and Economics: Meaning, rules of integration, standard results, Integration by parts, definite integration (basics only for doing problems of chapter 7 of Unit 1 of text) (Chapter 6 of unit I of text: Sections: 6.1, 6.2, 6.4, 6.10, 6.11)

Marginal cost, marginal revenue, Consumer's surplus, producer's surplus, consumer's surplus under pure competition, consumer's surplus under monopoly (Chapter 7 of unit I of text- Sections: 7.1, 7.2, 7.3, 7.4, 7.5)

Module 3 Index Numbers: Definition, types of index numbers, methods of construction of price index numbers, Laspeyer's price index number, Paasche's price index number, Fisher ideal index number, advantages of index numbers, limitations of index numbers

(Chapter 6 of Unit II of text- Sections: 6.1, 6.3, 6.4, 6.5, 6.6, 6.8, 6.16, 6.17)

Time series: Definition, Components of time series, Measurement of Trend (Chapter 7 of Unit II of text - Sections: 7.1, 7.2, 7.4)

TEXT: B M Aggarwal: Business Mathematics and Statistics Vikas Publishing House, New Delhi, 2009

REFERENCES:

1. Qazi Zameeruddin, et al : *Business Mathematics* , Vikas Publishing House, New Delhi, 2009

2. Alpha C Chicny, Kevin Wainwright: *Fundamental methods of Mathematical Economics* ,Mc-Graw Hill, Singapre, 2005

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours

Semester V

Actuarial Science (Open Course)

CODE: MM 1551.3 — Instructional hours per week: 3
No. of Credits: 2

Module 1 : Introduction to Insurance Business: What is Actuarial Science? Concept of Risk, Role of statistics in Insurance, Insurance business in India.

Introductory Statistics: Some important discrete distributions, Some important continuous distributions, Multivariate distributions

Module 2 : Feasibility of Insurance business and risk models for short terms: Expected value principle, Notion of utility,risk models for short terms

Future Lifetime distribution and Life tables: Future life time random variable, Curate future-life time, life tables, Assumptions for fractional ages, select and ultimate life tables.

Module 3 : Actuarial Present values of benefit in Life insurance products: Compound interest, Discount factor, Benefit payable at the moment of death, Benefit payable at the end of of year of death, relation between A and $\bar{A}$.

Annuities, certain, continuous life annuities, Discrete life annuities, Life annuities with m^{th}ly payments.

TEXT: Shylaja R. Deshmukh : Actuarial Statistics University press, Hyderabad, 2009. Chapters 1 - 6.

REFERENCES:

1. Bowers, Jr., N. L et al: *Actuarial Mathematics* , 2^{nd} Edition,The society of Actuaries, Illinois, Sahaumberg,1997
2. Palande, P. S. et al: *Insurance in India: Changing policies and Emerging Oppertunities* ,Response Books, New Delhi, 2003
3. Purohit, S. G. et al: *Statistics Using* R ,Narosa, New Delhi, 2008
4. www.actuariesindia.org

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 12 hours; Module 2: 21 hours; Module 3: 21 hours

Semester VI

Real Analysis II

CODE: MM 1641 Instructional hours per week: 5

No.of credits: 4

Text : R. G. Bartle, D. R. Sherbert, *Introduction to Real Analysis*, Third Edition, Wiley

This course builds on the first course in Real Analysis done earlier and concentrates on real valued functions. We discuss the three properties of continuity, differentiability and Riemann integrability. The history of how calculus developed must also be discussed (see `en.wikipedia.org/wiki/History_of_calculus`, for example).

Module 1

The intuitive geometric notion of continuity as an unbroken curve seen in the calculus course must be recalled and then the discussion should gradually lead to the ϵ-δ definition, as an effort to make this notion formal and rigorous. The connexion between continuity and existence of limit should be emphasized. The material contained in Sections 5.1–5.3 and Section 5.6 of the textbook forms the core of this part of the course. *Section 5.4,* UNIFORM CONTINUITY *and Section 5.5,* CONTINUITY AND GAUGES, *need not be discussed.*

Module 2

Differentiation and integration are extensively discussed in an earlier Calculus course, with a strong emphasis on computation. Here we take another look at differentiation from a conceptual point of view. It is based on Chapter 6 of the textbook. All the four sections of this chapter are to be discussed in detail.

Module 3

In this module, we discuss Riemann's theory of integration. It is based on Sections 7.1–7.3 of the text. *Section 7.4,* APPROXIMATE INTEGRATION *need not be discussed.*

Students have already seen integration as anti-differentiation and informally as the limit of sums in the calculus couse. All these idea are made more precise here. The historical evolution of the ideas leading to Riemann integral can be found in `en.wikipedia.org/wiki/Integral#History`. The differences between anti-differentiation and Riemann's theory of integration should be stressed. Section 7.3 of the textbook must be seen as establishing the links between anti-differntiation and Riemann integration, Examples 7.3.2(e) and 7.3.7(a), (b) are significant in this context.

References

1. A. D. Alexandrov et al., *Mathematics:Its Content, Methods and Meaning*, Dover

2. R. Dedekind, *Essays on The Theory of Numbers*, available as a freely dowloadable e-book at http://www.gutenberg.org/etext/21016)

3. W. Rudin, *Principles of Mathematical Analysis*, Second Edition, McGraw-Hill

4. A. E. Taylor, *General Theory of Functions and Integration*, Dover

Distribution of instructional hours:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

Semester VI

Linear Algebra

CODE: MM 1642 — Instructional hours per week: 4

No.of credits: 3

Text : Thoma Banchoff and John Wermer, *Linear Algebra Through Geometry*, Second Edition, Springer

In this course we introduce the basics of linear algebra and matrix theory with emphasis on their geometrical aspects. It is based on the Chapters 1–4 of the text.

Module 1

In this module we bring together some aspects of analytic geometry of two dimensions, solutions of simultaneous in two unknowns and theory of 2×2 matrices under the unified theme of linear transformations of the plane. It is based on Chapters 1 and 2 of the text.

Module 2

The ideas in the first module are extended to three dimensional space in this module. It is based on Chapter 3 of the text

Module 3

The concepts discussed in the first two modules are generalized to arbitrary dimensions in this module. It is based on Chapter 4 of the text.

TEXT: References:

1. T S Blyth and E F Robertson: *Linear Algebra*, Springer, Second Ed.
2. R Bronson and G B Costa: *Linear Algebra*, Academic Press, Seond Ed.
3. David C Lay: *Linear Algebra*, Pearson
4. K Hoffman and R Kunze: *Linear Algebra*, PHI

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

Semester VI

Complex Analysis II

Code: MM 1544 Instructional hours per week: 3

No.of credits: 3

Texts 1. Joseph Bak and Donald J. Newman, *Complex Analysis*. Third Edition, Springer

2. James Brown and Ruel Churchill, *Complex Variables and Applications*, Eighth Edition, McGraw-Hill

In this course, we consider some of the basic properties of functions analytic in a disc or on a punctured disc. It is based on parts Chapters 6, 9, 10, 11 of [1] and Chapters 6 and 7 of [2].

Module 1

We start with Sections 6.1 and 6.2 of [1]. In Section 6.1, *only the statement of* 6.5 Power Series Representation for Functions Analytic in a Disc *need be given; the proof need not be discussed*. But it should be linked to 2.10 Corollay to note that a function analytic in a disc is infinitely differentiable in it and with 2.11 Corollary to see how the coefficients of the series are related to the derivatives of the function. *Section 6.3 need not be discussed.*

We then pass on to a discussion of isolated singular points and residues, as in Chapter 6 (Sections 68–77). *Here and elsewhere, all examples and exercises involving logarithms must be skipped.*

Module 2

In this module, we consider the application of the Residue Theorem in the evaluation of some integrals. as in Chapter 7 of [2]. Only Sections 78–82 and Section 85 need be discussed. *Sections 83–84 and Sections 86–89 need not be considered.*

Section 11.2 of [1], Application of Contour Integral Methods to Evaluation and Estimation of Sums, must also be discussed, along with the relevant exercises in this section.

References:

1. Ahlfors, L. V, *Copmlex Analysis*, McGraw-Hill, 1979.

2. J M Howie, *Complex Analysis*, Springer

Distribution of instructional hours:

Module 1: 27 hours; Module 2: 27 hours

Semester VI

Abstract Algebra II

CODE: MM 1544 Instructional hours per week: 3

No.of credits: 3

TEXT: John B. Fraleigh, *A First Course in Abstract Algebra*. Seventh Edn, Narosa

In this course, we discuss more of group theory and also the basics of ring theory. It is based on parts of Chapters II–V of the text. As in the first course, due emphasis must be given to problem solving.

Module 1

In this part of the course, we discuss homomorphism of groups and factor groups, as in Sections 13–15 of the text. *The last two parts of Section 15,* SIMPLE GROUPS *and* THE CENTER AND COMMUTATOR SUBGROUPS *need not be discussed.*.

Module 2

We start by recalling the definition of rings, seen in an earlier course on algebra. Then Sections 18-20 must be discussed in detail. *Sections 21–25 need not be discussed*, But Section 26 on homomorphisms and factor rings must be done in full.

REFERENCES:

1. I N Herstein, *Topics in Algebra*, Vikas Publications
2. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications
3. D A R Wallace, *Groups, Rings and Fields*, Springer

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours

Semester VI

Computer Programming

CODE: MM 1541 | Instructional hours per week: 5

No.of credits: 4

In this course, we teach document preparation in computers using the LaTeX typesetiing program and also the basics of computer programming using Python. Since the operatig system to be used is GNU/Linux, fundamentals of this OS are also to be discussed.

Module 1

Text : Matthias Kalle Dalheimer and Matt Welsh, *Running Linux*, Fifth Edition, O'Reilly

In this module, we consider the fundamentals of the GNU/Linux operating system. It is based on Chapter 4, BASIC UNIX COMMANDS AND CONCEPTS, of the text. Students should be taught about the Linux directory structure and the advantages of keeping their files in well structured directories. Since they will be using the command line interface most of the time, this entails facility in using such commands as `mkdir`, `pwd`, `cd`, `ls`, `cp`, `mv`, `ls` and so on.

Module 2

Text : *LaTeX Tutorials—A Primer* by Indian TeX Users Group

In this module, we discuss computer typesetting using LaTeX, Chapters 1–2 of the text must be discussed in full. On bibliography, only the first section of Chapter 3 need be discussed. Also, only the first section of Chapter 4—on table of conetents–need be done. Chapters 6–9 are to be done in full. Finally Chapter 12 also is to be discussed in full.

Module 3

Text : Vernon L. Ceder, *The Quick Python Book*, Second Edition, Manning

It is based on Chapters 3–9 of the text. The concepts in Chapters 3–8 must be discussed in full, but in Chapter 9, only Sections 9.1–9.5 need be discussed.

The programs done in class should all have a mathematical content. SOme possibilities are listed below:

- Factorial of a number

- Checking primality of a number

- Listing all primes below a given number
- Prime factorization of a number
- Finding all factors of a number
- GCD of two numbers using the Euclidean Algorithm
- Finding the multiples in Bezout's Identity

Distribution of instructional hours:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

Semester VI

Graph Theory (Elective)

CODE: MM 1661.1 — Instructional hours per week: 3

No. of credits: 3

Overview of the Course: The course has been designed to build an awareness of some of the

fundamental concepts in Graph Theory and to develop better understanding of the subject so as to use these ideas skillfully in solving real world problems.

Module 1 A brief history of Graph Theory: The Königsberg bridge problem, the history of the Four Colour Theorem for maps, Contributions to Graph Theory by Euler, Kirchoff, Cayley, Mobius, De Morgan, Hamilton, Erdös, Tutte, Harary, etc. (A maximum of three hours may be allotted to this sub-module. In addition to sections 1.2 and 1.6 of the text, materials for this part can be had from other sources including the internet.)Graphs: Definition of graph, vertex, edge, incidence, adjacency, loops, parallel edges, simple graph. Representation of graphs, diagrammatic representation, matrix representation (adjacency* matrix and incidence matrix only). Finite and infinite graphs, Definition of directed graphs, illustrative examples, Directed graphs, Applications of graphs. [sections 1.1, 1.2, 1.3, 1.4, 7.1, 9.1, 9.2]Degree of a vertex, odd vertex, even vertex, relation between sum of degrees of vertices and the number of edges in a graph, and its consequence: number of odd vertices in a graph is even. Isolated vertex, pendant vertex, null graph, complete graphs [page 32], bipartite graphs [page 168], complete bipartite graph [page 192-prob 8.5], regular graph, complement* of a graph, graph isomorphisms, self complementary* graphs, illustrative examples. [sections 1.4, 1.5, 2.1]Sub-graphs, edge disjoint sub-graphs, spanning sub-graphs*, induced subgraphs [sections 2.2]The decanting problem and its graph model [no solution at this point]. The puzzle with multicolour cubes [problem 1.8 and section 2.3].

Module 2 Walks, open walks, closed walks, paths, circuits, end vertices of a path, path joinig two vertices, length of a path, connected and disconnected graphs. Components of a graph. [sections 2.4, 2.5]Euler line, Euler graph, unicursal line, unicursal graph, characterisaion of Euler graph, Concept of Euler digraph [section 2.5, 9.5], Solution of the decanting problem. The Königsberg problem, the Chinese postman problem* and the Teleprinter's problem, their graph models and solutions. [problem 1.8 and sections 2.3, 1.2, 9.5]

Module 3 Trees- properties of trees, distance, eccentricity, center, radius, diameter, spanning tree, illustrative examples. [sections 3.1, 3.2, 3.3, 3.4, 3.7]Planar graphs examples of planar and non-planar graphs, different representations of a planar graph. Regular polyhedra, Euler's polyhedral formula. [Theorem 5.6, without proof] .

Illustrative examples, Kuratowski's graphs and their importance in the theory of planar graphs, forbidden sub-graph,

characterisaion of planar graph [Kuratowski's theorem, Theorem 5.9, without proof], illustrative examples-both planar and non-planar. [sections 5.2, 5.3, 5.4, 5.5] Graph theoretic version of the Four Colour Theorem, without proof.

TEXT: Narsingh Deo: Graph Theory with applications for Engineering and Computer Science, Prentice Hall of India Pvt. Ltd., 2000.
References:

1. BalakrishnanR and Ranganatahan: *A Text Book of Graph Theory*, Springer
2. Body J A and Murthy U S R:*Graph Theory with Applications*, The Macmillan Press
3. Harary F: *Graph Theory*, Addison-Wesley
4. Vasudev C: *Graph Theory with Applications*
5. West D B: *Introduction to Graph Theory*, Prentice Hall of India Pvt. Ltd.

Note: Generally, the references are from NARSINGH DEO. Those marked with an asterisk are found elsewhere.DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours

Semester VI

Fuzzy Mathematics (Elective)

CODE: MM 1661.2

Instructional hours per week: 3
No. of credits: 2

Module 1 FROM CRISP SETS TO FUZZY SETS: A PARADIGM SHIFT.Introduction-crisp sets: an overview-fuzzy sets: basic types and basic concepts of fuzzy sets, Fuzzy sets versus crisp sets, Additional properties of cuts, Representation of fuzzy sets.

Module 2 OPERATIONS ON FUZZY SETS AND FUZZY ARITHMETIC:Operations on fuzzy sets-types of operations, fuzzy complements, fuzzy intersections, t-norms, fuzzy unions, t-conorms.

Fuzzy numbers, Linguistic variables, Arithmetic operations on intervals, Arithmetic operations on fuzzy numbers.

Module 3 FUZZY RELATIONS :Crisp versus fuzzy relations, projections and cylindric extensions, Binary fuzzy relations, Binary relations on a single set, Fuzzy equivalence relations.

TEXT: George J Klir and Yuan: Fuzzy sets and fuzzy logic: Theory and applications, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
Chapter 1: Sections 1.1 to 1.4
Chapter 2: Sections 2.1 and 2.2
Chapter 3: Sections 3.1 to 3.4
Chapter 4: Sections 4.1 to 4.4
Chapter 5: Sections 5.1 to 5.5

References:

1. Klir G J and T Folger: *Fuzzy sets, Uncertainty and Information*, PHI Pvt.Ltd., New Delhi, 1998

2. H J Zimmerman: *Fuzzy Set Theory and its Applications*, Allied Publishers, 1996.

3. Dubois D and Prade H: *Fuzzy Sets and Systems: Theory and Applications*, Ac.Press, NY, 1988.

Distribution of instructional hours:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours

Semester VI

Mechanics (Elective)

CODE: MM 1661.3 | Instructional hours per week: 3
No. of credits: 2

Part A: STATICS

Module 1 Introduction, composition and resolution of forces, parallelogram law of forces, triangle law of forces, Lami's theorem, polygon of forces, $\lambda-\mu$ theorem, resultant of a finite number of coplanar forces acting upon a particle, conditions of equilibrium, parallel forces, resultant of two parallel forces acting upon a rigid body, moments, moments of a force about a point and about an axis, generalized theorem of moments.

Module 2 Couples, equilibrium of a rigid body acted on by three coplanar forces, general conditions of equilibrium of a rigid body under coplanar forces, friction, laws of friction, limiting friction, coefficient of friction and simple problems.

Part B : DYNAMICS

Module 3 Velocity, relative velocity, acceleration, parallelogram laws of acceleration, motion under gravity, Newtons laws of motion and their applications to simple problems. Impulse, work, energy, kinetic and potential energies of a body, principle of conservation of energy.

Module 4 Projectiles, Range on an inclined plane, Collision of elastic bodies, Newton's experimental law, Impact of sphere on a plane, Direct and oblique impact of two spheres, Loss of kinetic energy by impact, Simple harmonic motion, Examples of simple harmonic motion, Simple pendulum.

TEXT: by S.L. Loney, The Elements of Statics and Dynamics, Part-I and Part-II, AITBSPublications and distributions (Regd), Delhi
Distribution of instructional hours:

Module 1: 15 hours; Module 2: 12 hours; Module 3: 15 hours, Module 4: 12 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Physics

Semester I

Mathematics-I
(Differentiation and Analytic Geometry)
Code: MM 1131.1

Instructional hours per week: 4 No. of Credits:3

Overview
The complementary course intended for Physics students lays emphasis on the application of mathematical methods to Physics. The two modules on Calculus links the topic to the real world and the student's own experience as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. Analytic geometry presented here is important in applications of calculus.

Module 1: Differentiation with applications to Physics-I

- Functions and graphs of functions with examples from Physics. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid and Brachistochrone problem.
  Exercise set 1.8; Questions 31 - 34, 37 and 39.

- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits.
  Exercise set 2.1; Questions 27 and 28.

- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.
  Exercise set 3.1; Questions 1 - 4 and 15, 16, 18 - 21.
  Exercise set 3.2; Question 39.

- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates. Local linear approximation. Differentials.
  Examples 1 - 6.
  Exercise set 3.3; Question 68.
  Exercise set 3.4; Question 32.
  Exercise set 3.8; Questions 57 - 60.

- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.
  Examples 1 - 7. Exercise set 4.4; Questions 8, 9, 23, 27, 30 - 32.

- Absolute maxima and minima. Applied maximum anmd minimimum problems.
  Exercise set 4.6; Questions 47, 48, 56, 59.

- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems 4.1.2 and 4.83 (consequences of the Mean Value Theorem).
  Exercise set 4.8; Questions 22 - 25.
- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function. Graphs and applications involving logarithmic and exponential functions. Logistic curves. Example 4 of section 7.4 (Newton's Law of Cooling).
  Exercise set 7.4; Questions 31, 35, 49 - 50.
- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hanging cables and other applications. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.
  Exercise set 7.8; Questions 69 and 72.

Module 2: Differentiation with applications to Physics-II

- Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximating functions by polynomials. Taylor series and Maclaurin series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.
- Functions of several variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and differentials. The chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians.
  Exercise set 14.3; Questions 47 and 48.
  Exercise set 14.4; Questions 49 and 50.
  Exercise set 14.5; Questions 41. 42 and 46.
- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.
  Exercise set 14.9; Question 20.

Module 3: Analytic Geometry

- Geometric definition of a conic-the focus, directrix and eccentricity of a conic. Classification of conics into ellipse, parabola and hyperbola based on the value of eccentricity. Sketch of the graphs of conics. Reflection properties of conic sections.
  Exercise set 11.4; Questions 39 - 43.

- Equations of the conics in standard positions. Equations of the conics which are translated from standard positions vertically or horizontally. Parametric representation of conics in standard form. Condition for a given straight line to be a tangent to a conic. Equation of the tangent and normal to a conic at a point.

- Asymptotes of a hyperbola. Equation of the asymptotes.

- Conic sections in polar coordinates. Eccentricity of an ellipse as a measure of flatness. Polar equations of conics. Sketching conics in polar coordinates. Kepler's Laws.

  Example 4 of section 11.6.

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Semester II

Mathematics-II
(Integration and Vectors)

Code: MM 1231.1

Instructional hours per week: 4 No. of Credits: 3

Overview

The complementary course in the second semester continues the trend indicated in the first, namely, laying emphasis on applications of integral calculus and vectors to problems in Physics. Module 1 consists of various applications of integration techniques. It also covers multiple integrals. Modules 2 and 3 deal with vector calculus and its applications in detail.

Module 1: Applications of integration

- Integral curves, integration from the view point of differential equations, direction fields
  Exercise set 5.2; Questions 43, 44 and 51.

- Rectilinear motion: finding position and velocity by integration. Uniformly accelerated motion. The free-fall model. integrating rates of change. Displacement in rectilinear motion. Distance travelled in rectilinear motion. Analysing the velocity versus time curve. Average value of a continuous function. Average velocity revisited.
  Exercise set 5.7; Questions 3, 4, 5, 6, 29, 39, 45 and 55.

- Use of definite integrals in finding area under curves, area between two curves, volume of revolution, arc length and surface area of a solid of revolution.

- The idea of approximating the volume under a bounded surface in 3-space by volumes of boxes, leading to the definition of double integrals of functions of two variables over bounded regions. Evaluation of double integrals by iterated integrals. Evaluation by changing to polar co-ordinates and by suitably changing order of integration in the iterated integral. Applications to finding the volume of solids under bounded surfaces.

- Triple integrals over bounded regions in three space. Evaluation by iterated integrals. Cylindrical coordinates and spherical coordinates and their relation to Cartesian coordinates. Use of cylindrical and spherical co-ordinates in evaluating triple integrals. Applications of triple integrals to finding volumes of solid objects.

Module 2: Vector Differentiation

- Vector function of a single variable and representation in terms of standard basis. Limit of a vector function and evaluation of limit in Cartesian representation. Continuous vector functions and the idea that such functions represent oriented space curves. Examples.
- Derivative of a vector function and its geometric significance. Derivative in terms of Cartesian components. Tangent vector to a curve, smooth and piecewise smooth curves. Applications to finding the length and curvature of space curves, velocity and acceleration of motion along a curve etc.
- Scalar field and level surfaces. The gradient vector of a scalar field (Cartesian form) at a point and its geometric significance. Gradient as an operator and its properties. Directional derivative of a scalar field and its significance. Use of gradient vector in computing directional derivative.
- Vector fields and their Cartesian representation. Sketching of simple vector fields in the plane. The curl and divergence of a vector field(Cartesian form) and their physical significance. The curl and divergence as operators, their properties. Irrotational and solenoidal vector fields. Various combinations of gradient, curl and divergence operators.

Module 3: Vector Integration

- The method of computing the work done by a force field in moving a particle along a curve leading to the definition of line integral of a vector field along a smooth curve. Scalar representation of line integral. Evaluation as a definite integral. Properties. Line integral over piecewise smooth curves. Green's theorem in the plane (without proof) for a region bounded by a simple closed piecewise smooth curve.
- Oriented surfaces. The idea of flux of a vetor field over a surface in 3-space. The surface integral of a vector field over a bounded oriented surface. Evaluation by reducing to a double integral. Use of cylindrical and spherical co-ordinates in computing surface integral over cylindrical and spherical surfaces.
- Stokes' theorem (without proof) for an open surface with boundary a piecwise smooth closed curve. Gauss' divergence theorem (without proof). Verification of the theorems in simple cases and their use in computing line integrals or surface integrals which are difficult to evaluate directly. Physical intrepretation of divergence and curl in terms of the velocity field of a fluid flow.
- Conservative fields and potential functions. Relation of conservative vector fields to their irrotational nature and the path- independence of line integrals in the field (without proof). Significance of these results in the case of conservative force fields such as gravitational, magnetic and electric fields. Method of finding the potential function of a conservative field.

Text : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Semester III

Mathematics-III
(Differential Equations, Theory of Equations and Theory of Matrices)

Code: MM 1331.1

Instructional hours per week:5 No. of Credits: 4

Module 1: Differential equations

- Review of basic concepts about differential equations and their solutions. Method of solving special types of first order ODEs such as variable separable, exact, homogeneous, and linear. Finding the family of curves orthogonal to a given family.
- Second order linear differential equations. Nature of the general solution of homogeneous and non-homogeneous linear ODEs. Extension to higher order ODE.
- Second order linear homogeneous ODEs with constant coefficients. The characteristic equation and its use in finding the general solution. Extension of the results to higher order ODEs.
- Second order linear non-homogeneous ODEs with constant coefficients. General solution as the sum of complementary function and particular integral. Second order linear differential operator and its properties. The inverse operator and its properties. Operator method for finding the particular integral of simple functions. Extension of the results to higher order equations. Cauchy and Legendre equations and their solutions by reducing to equations with constant coefficients by suitable change of variable.

Module 2: Linear Algebra

- The rows and columns of a matrix as elements of $\mathbb{R}^n$ for suitable n. Rank of a matrix as the maximum number of linearly independent rows/columns. Elementary row operations. Invariance of rank under elementary row operations. The echelon form and its uniqueness. Finding the rank of a matrix by reducing to echelon form.
- Homogeneous and non-homogeneous system of linear equations. Results about the existence and nature of solution of a system of equations in terms of the ranks of the matrices involved.
- The eigen value problem. Method of finding the eigen values and eigen vectors of a matrix. Basic properties of eigen values and eigen vectors. Eigen values and eigen vectors of a symmetric matrix.
- Diagonalisable matrices. Advantages of diagonalisable matrices in computing matrix powers and solving system of equations. The result(without proof) that a square matrix of order n is diagonalisabe (i) if and only if it has n linearly independent eigen vectors (ii) if it has n distinct eigen values. Method of diagonalising a matrix. Diagonalisation of real symmetric matrices. Similar matrices.

Module 3: Theory of Equations

- Fundamental theorem of Algebra (without proof), relations between roots and coefficients of a polynomial, finding nature of roots of polynomials without solving-Des Cartes' rule of signs, finding approximate roots via bisection method, Newton-Raphson method

Text for Module 1: Kreyzig, Advanced Engineering Mathematics, 8^{th} edition, John Wiley.
Text for Module 2:Peter V. O' Neil, Advanced Engineering Mathematics, Thompson Publications, 2007
Text for Module 3: Barnard and Child, Higher Algebra, Macmillan
Advanced Engineering Mathematics, K A Stroud, 4th Edition, Palgrave, 2003

Semester IV

Mathematics-IV
(Complex Analysis, Fourier Series and Fourier Transforms)

Code: MM 1431.1

Instructional hours per week: 5 No. of Credits: 4

Module 1: Complex Analysis

- Representation of complex numbers, operations involving them, conjugates, polar form of complex numbers, De-Moivre's formula, complex number sets and functions, their limit, continuity, derivatives. Analytic functions, Cauchy-Riemann equations and Laplace equation, harmonic functions, proof that an analytic function with constant modulus is constant, exponential, trigonometric, hyperbolic,logarithmic functions in $\mathbb{C}$
- Complex integration: Line integral (definition only, proof on existence not required), section on bounds on line integrals may be omitted, Cauchy's integral theorem and formula, and problems involving them, connected, multiply connected domains, Cauchy's inequality, Liouville's theorem, Morera's theorm (all without proof), problems using the theorems
- Complex sequences, series, their convergence tests, problems using the tests, power series and their convergence, radius of convergence of power series, addition, multiplication of power series, power series representation of analytic functions, Taylor, MacLaurin's series approximations, problems to find the series representations of important functions
- Laurent series of functions, its singularities, poles, and zeros, Cauchy's resdue integration method, findin residues, residue theorem (without proof), problems and applications using it

Module 2: Fourier series and transforms

- Periodic functions, trigonometric series, Fourier series, evaluation of Fourier coefficients for functions defined in $(-\infty, +\infty)$, Fourier series for odd and even functions, half range series, Fourier series for odd and even functions, Fourier series of functions defined in $(-L, +L)$.
- Fourier integrals and Fourier transforms.

Text:Kreyzig, Advanced Engineering Mathematics, 8^{th} edition, John Wiley.

References

1. Thomas and Finney, Calculus and Analytic Geometry, Ninth Edition, Addison–Wesley.
2. Michael D. Greenberg, Advanced Engineering Mathematics, Pearson Education, 2002.
3. James Stewart, Essential Calculus, Thompson Publications, 2007.
4. David C. Lay, Linear Algebra, Thompson Publications, 2007.
5. George F Simmons, Differential equations with applications and historical notes, Tata McGraw Hill, 2003
6. T. Gamelin, Complex Analysis, Springer-verlag, 2006
7. Brown and Churchil, Complex Variables and Applications, McGraw-Hill Higher Education; 8 edition, 2008
8. S L Loney, The elements of coordinate geometry
9. SAGE Math official website http://www.sagemath.org/
10. Gnuplot official website containing documentation and lot of examples http://www.gnuplot.info/
11. More help and examples on gnuplot http://people.duke.edu/ hpgavin/gnuplot.html
12. Maxima documentations http://maxima.sourceforge.net/documentation.html

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Chemistry

Semester I

Mathematics-I
(Differentiation and Matrices)
Code: MM 1131.2

Instructional hours per week: 4 No. of Credits:3

Overview
The complementary course intended for Chemistry students lays emphasis on the application of mathematical methods to Chemistry. The two modules on Calculus links the topic to the real world and the student's own experience as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. The third module covers matrix theory.

Module 1: Differentiation with applications to Chemistry-I

- Functions and graphs of functions with examples from Chemistry. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid and Brachistochrone problem.
  Exercise set 1.8; Questions 31 - 34.
- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits.
  Exercise set 2.1; Questions 27 and 28.
- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.
  Exercise set 3.1; Questions 1,2 and 16.
- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates. Local linear approximation. Differentials.
  Exercise set 3.8; Questions 53 - 55.
- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.
  Examples 1 - 7. Exercise set 4.4; Questions 8, 9, 30 - 32.
- Absolute maxima and minima. Applied maximum and minimimum problems.
  Exercise set 4.6; Questions 47 and 48.
- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems 4.1.2 and 4.83 (consequences of the Mean Value Theorem).

- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function. Graphs and applications involving logarithmic and exponential functions.
  Exercise set 7.4; Question 50.

- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.

Module 2: Differentiation with applications to Chemistry-II

- Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximating functions by polynomials. Taylor series and Maclaurin series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.

- Functions of several variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and differentials. The chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians.
  Exercise set 14.3; Questions 47 and 48.
  Exercise set 14.4; Question 50.
  Exercise set 14.5; Question 42.

- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.
  Exercise set 14.9; Question 20.

Module 3 : Linear Algebra

- The rows and columns of a matrix as elements of $\mathbb{R}^n$ for suitable n. Rank of a matrix as the maximum number of linearly independent rows/columns. Elementary row operations. Invariance of rank under elementary row operations. The echelon form and its uniqueness. Finding the rank of a matrix by reducing to echelon form.

- Homogeneous and non-homogeneous system of linear equations. Results about the existence and nature of solution of a system of equations in terms of the ranks of the matrices involved.

- The eigen value problem. Method of finding the eigen values and eigen vectors of a matrix. Basic properties of eigen values and eigen vectors. Eigen values and eigen vectors of a symmetric matrix.

- Diagonalisable matrices. Advantages of diagonalisable matrices in computing matrix powers and solving system of equations. The result(without proof) that a square matrix of order n is diagonalisabe (i) if and only if it has n linearly independent eigen vectors (ii) if it has n distinct eigen values. Method of diagonalising a matrix. Diagonalisation of real symmetric matrices. Similar matrices.

Text for Module 1 and 2 : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Text for Module 3 : Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007

Semester II

Mathematics-II
(Integration, Differential Equations and Analytic Geometry)

Code: MM 1231.2

Instructional hours per week: 4 No. of Credits: 3

Overview

The complementary course in the second semester continues the trend indicated in the first, namely, laying emphasis on applications of integral calculus and vectors to problems in Chemistry. Module 1 consists of various applications of integration techniques. It also covers multiple integrals. Modules 2 deals with differential equations while Module 3 covers analytic geometry.

Module 1: Applications of integration

- Integral curves, integration from the view point of differential equations, direction fields
  Exercise set 5.2; Questions 43 and 44.
- Rectilinear motion: finding position and velocity by integration. Uniformly accelerated motion. The free-fall model. Integrating rates of change. Displacement in rectilinear motion. Distance travelled in rectilinear motion. Analysing the velocity versus time curve. Average value of a continuous function. Average velocity revisited.
  Exercise set 5.7; Questions 3, 4, 5, 6, 29 and 55.
- Use of definite integrals in finding area under curves, area between two curves, volume of revolution, arc length and surface area of a solid of revolution.
- The idea of approximating the volume under a bounded surface in 3-space by volumes of boxes, leading to the definition of double integrals of functions of two variables over bounded regions. Evaluation of double integrals by iterated integrals. Evaluation by changing to polar co-ordinates and by suitably changing order of integration in the iterated integral. Applications to finding the volume of solids under bounded surfaces.

- Triple integrals over bounded regions in three space. Evaluation by iterated integrals. Cylindrical coordinates and spherical coordinates and their relation to Cartesian coordinates. Use of cylindrical and spherical co-ordinates in evaluating triple integrals. Applications of triple integrals to finding volumes of solid objects.

Module 2: Differential equations

- Review of basic concepts about differential equations and their solutions. Method of solving special types of first order ODEs such as variable separable, exact, homogeneous, and linear. Finding the family of curves orthogonal to a given family.
- Second order linear differential equations. Nature of the general solution of homogeneous and non-homogeneous linear ODEs. Extension to higher order ODE.
- Second order linear homogeneous ODEs with constant coefficients. The characteristic equation and its use in finding the general solution. Extension of the results to higher order ODEs.
- Second order linear non-homogeneous ODEs with constant coefficients. General solution as the sum of complementary function and particular integral. Second order linear differential operator and its properties. The inverse operator and its properties. Operator method for finding the particular integral of simple functions. Extension of the results to higher order equations. Cauchy and Legendre equations and their solutions by reducing to equations with constant coefficients by suitable change of variable.

Module 3: Analytic Geometry

- Geometric definition of a conic-the focus, directrix and eccentricity of a conic. Classification of conics into ellipse, parabola and hyperbola based on the value of eccentricity. Sketch of the graphs of conics. Reflection properties of conic sections.
  Exercise set 11.4; Questions 39 - 43.
- Equations of the conics in standard positions. Equations of the conics which are translated from standard positions vertically or horizontally. Parametric representation of conics in standard form. Condition for a given straight line to be a tangent to a conic. Equation of the tangent and normal to a conic at a point.
- Asymptotes of a hyperbola. Equation of the asymptotes.
- Conic sections in polar coordinates. Eccentricity of an ellipse as a measure of flatness. Polar equations of conics. Sketching conics in polar coordinates. Kepler's Laws.

  Example 4 of section 11.6.

Text for Module 1 and 3 : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Text for Module 2 : Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition,John Wiley.

Semester III

Mathematics-III
(Vector Analysis and Theory of Equations)

Code: MM 1331.12

Instructional hours per week:5 No. of Credits: 4

Module 1: Vector Differentiation

- Vector function of a single variable and representation in terms of standard basis. Limit of a vector function and evaluation of limit in Cartesian representation. Continuous vector functions and the idea that such functions represent oriented space curves. Examples.
- Derivative of a vector function and its geometric significance. Derivative in terms of Cartesian components. Tangent vector to a curve, smooth and piecewise smooth curves. Applications to finding the length and curvature of space curves, velocity and acceleration of motion along a curve etc.
- Scalar field and level surfaces. The gradient vector of a scalar field (Cartesian form) at a point and its geometric significance. Gradient as an operator and its properties. Directional derivative of a scalar field and its significance. Use of gradient vector in computing directional derivative.
- Vector fields and their Cartesian representation. Sketching of simple vector fields in the plane. The curl and divergence of a vector field(Cartesian form) and their physical significance. The curl and divergence as operators, their properties. Irrotational and solenoidal vector fields. Various combinations of gradient, curl and divergence operators.

Module 2: Vector Integration

- The method of computing the work done by a force field in moving a particle along a curve leading to the definition of line integral of a vector field along a smooth curve. Scalar representation of line integral. Evaluation as a definite integral. Properties. Line integral over piecewise smooth curves. Green's theorem in the plane (without proof) for a region bounded by a simple closed piecewise smooth curve.
- Oriented surfaces. The idea of flux of a vetor field over a surface in 3-space. The surface integral of a vector field over a bounded oriented surface. Evaluation by reducing to a double integral. Use of cylindrical and spherical co-ordinates in computing surface integral over cylindrical and spherical surfaces.
- Stokes' theorem (without proof) for an open surface with boundary a piecwise smooth closed curve. Gauss' divergence theorem (without proof). Verification of the theorems in simple cases and their use in computing line integrals or surface integrals which are difficult to evaluate directly. Physical intrepretation of divergence and curl in terms of the velocity field of a fluid flow.

- Conservative fields and potential functions. Relation of conservative vector fields to their irrotational nature and the path- independence of line integrals in the field (without proof). Significance of these results in the case of conservative force fields such as gravitational, magnetic and electric fields. Method of finding the potential function of a conservative field.

Module 3: Theory of Equations

- Fundamental theorem of Algebra (without proof), relations between roots and coefficients of a polynomial, finding nature of roots of polynomials without solving-Des Cartes' rule of signs, finding approximate roots via bisection method, Newton-Raphson method

Text for Module 1 and 2 : Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

Text for Module 3 :
Barnard and Child, *Higher Algebra*, Macmillan

K A Stroud, *Advanced Engineering Mathematics*, 4^{th} edition,Palgrave, 2003.

Semester IV

Mathematics-IV
(Abstsract Algebra and Linear Transformations)

Code: MM 1431.2

Instructional hours per week: 5 No. of Credits: 4

Module 1: Abstract Algebra

- Groups–definition and examples, elementary properties, finite groups and subgroups, cyclic groups, elementary properties, groups of permutations

- Rings and Fields - definition and examples

  [Sections 2, 4, 5, 6, 8 (excluding the subsection on Cayley's theorem) and 18 (excluding the subsection on homomorphism and isomorphism) of text . Proofs of theorems are excluded. However ideas contained in theorems and definitions should be explained with illustrative examples and problems.]

  (See also J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications for examples of symmetry groups)]

Module 2: Linear Transformations

- Linear independence of vectors. Linear independence of Matrix columns.

- Linear transformations from $\mathbb{R}^n$ into $\mathbb{R}^m$. Matrix transformations. Linear transformation.
- The matrix of a Linear transformation. Matrix representation of simple tranformations such as rotation, reflection, projection etc. on the plane.

[Sections 1.7, 1.8, and 1.9 of text]

Text for Module 1: J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications
Text for Module 2: David C. Lay, *Linear Algebra and its applications*, Third Edition Pearson

References

1. Thomas and Finney, Calculus and Analytic Geometry, Ninth Edition, Addison–Wesley.
2. Michael D. Greenberg, Advanced Engineering Mathematics, Pearson Education, 2002.
3. James Stewart, Essential Calculus, Thompson Publications, 2007.
4. David C. Lay, Linear Algebra, Thompson Publications, 2007.
5. George F Simmons, Differential equations with applications and historical notes, Tata McGraw Hill, 2003
6. T. Gamelin, Complex Analysis, Springer-verlag, 2006
7. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications
8. Brown and Churchil, Complex Variables and Applications, McGraw-Hill Higher Education; 8 edition, 2008
9. S L Loney, The elements of coordinate geometry
10. SAGE Math official website http://www.sagemath.org/
11. Gnuplot official website containing documentation and lot of examples http://www.gnuplot.info/
12. More help and examples on gnuplot http://people.duke.edu/ hpgavin/gnuplot.html
13. Maxima documentations http://maxima.sourceforge.net/documentation.html

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Geology

Semester I
Mathematics-I
(Differentiation and Theory of Equations)

CODE: MM 1131.3

Instructional hours per week: 4
No. of Credits: 3 credits

Overview of the course:
The complementary course intended for Geology students lays emphsis on the application of mathematical methods to Geology. The two modules on Calculus links the topic to *the real world and the student's own experience* as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. Applications to Geology on the lines of those in Physics as given in the text could be obtained from the net. The third module covers theory of equations.

Module 1: Differentiation with applications to Geology-I

- Functions and graphs of functions with examples from Geology. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid.
  Exercise set 1.8; Questions $31 - 34$.

- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits. Indeterminate forms of the type $0/0$.
  Exercise set 2.1; Questions 27 and 28.

- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.
  Exercise set 3.1; Questions 1, 2 and 16.

- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates.Local linear approximation. Differentials.
  Examples $1 - 6$.Exercise set 3.8; Questions $53 - 55$.

- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.
  Examples $1 - 7$. Exercise set 4.4; Questions 8, 9, $30 - 32$.

- Absolute maxima and minima. Applied maximum anmd minimimum problems.
  Exercise set 4.6; Questions 47 and 48.

- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems $4.1.2$ and 4.83 (consequences of the Mean Value Theorem).

- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function.Graphs and applications involving logarithmic and exponential functions.
  Exercise set 7.4; Question 50.

- L'Hospital's Rule for finding the limits (without proof) of indeterminate forms of the type $0/0$ and ∞/∞. Analysing the growth of exponential functions using L'Hospital's Rule. Indeterminate forms of type$0 \cdot \infty$ and $\infty - \infty$ and their evaluation by converting them to $0/0$ or ∞/∞ types. Indeterminate forms of type 0^0, ∞^0 and 1^∞.

- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.

Module 2: Differentiation with applications to Geology-II

- Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximatingfunctions by polynomials. Taylor series and Maclaurin's series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.

- Functions of two variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians. Exercise set 14.3; Questions 47 and 48.Exercise set 14.4; Question 50.Exercise set 14.5; Question 42.

- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.
  Exercise set 14.9; Question 20.

Module 3: Theory of equations

- Polynomial equations and fundamental theorem of algebra (without proof). Applications of the fundamental theorem to equations having one or more complex roots, rational roots or multiple roots.

- Relations between roots and coefficients of a polynomial equation and computation of symmetric functions ofroots. Finding equations whose roots are functions of the roots of a given equation. Reciprocal equation and method of finding its roots.

- Analytical methods for solving polynomial equations of order up to four-quadratic formula, Cardano's method for solving cubic equations), Ferrari's method (for quartic equations). Remarks about the insolvability of equations of degree five or more. Finding the nature of roots without solving-Des Cartes' rule of signs.

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

TEXTS:

1. Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
2. Barnard and Child, *Higher Algebra*, Macmillan.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Geology

Semester II
Mathematics-II
(Integration, Differential Equations and Matrices)

Code: MM 1231.3 — Instructional hours per week: 4
No. of Credits: 3

Overview of the course:
The complementary course in the second semester continues the trend indicated in the first, namely, laying emphasis on applications of integral calculus and vectors to problems in Geology. Module 1 consists of a review of basic integration techniques and the applications of integration. It also covers multiple integrals. Module 2 deals with differential equations, while Module 3 covers matrix theory.

Module 1: Integration (with applications to Geology)

- Indefinite integrals (Review only), integral curves, integration from the view point of differential equations, direction fields
  Exercise set 5.2; Questions 43 and 44
- (Review only) Definite integral and Fundamental Theorem of Calculus.
- Rectilinear motion: finding position and velocity by integration. Uniformly accelerated motion. The free-fall model. integrating rates of change. Displacement in rectilinear motion. Distance travelled in rectilinear motion. Analysing the velocity versus time curve. Average value of a continuous function. Average velocity revisited.
  Exercise set 5.7; Questions 3, 4, 5, 6, 29 and 55
- Review of integration techniques.
- Use of definite integrals in finding area under curves, area between two curves, volume of revolution, arc length and surface area of a solid of revolution.
- The idea of approximating the volume under a bounded surface in 3-space by volumes of boxes, leading to the definition of double integrals of functions of two variables over bounded regions. Evaluation of double integrals by iterated integrals. Evaluation by changing to polar co-ordinates and by suitably changing order of integration in the iterated integral. Applications to finding the volume of solids under bounded surfaces.
- Triple integrals over bounded regions in three space. Evaluation by iterated integrals. Cylindrical coordinates and spherical coordinates and their relation to Cartesian coordinates. Use of cylindrical and spherical co-ordinates in evaluating triple integrals. Applications of triple integrals to finding volumes of solid objects.

Module 2: Differential Equations

- Review of basic concepts about differential equations and their solutions. Method of solving special types of first order ODEs such as variable separable, exact, homogeneous, and linear. Finding the family of curves orthogonal to a given family.

- Second order linear differential equations. Nature of the general solution of homogeneous and non-homogeneous linear ODEs. Extension to higher order ODEs.

- Second order linear homogeneous ODEs with constant coefficients. The characteristic equation and its use in finding the general solution. Extension of the results to higher order ODEs.

- Second order linear non-homogeneous ODEs with constant coefficients. General solution as the sum of complementary function and particular integral. Second order linear differential operator and its properties. The inverse operator and its properties. Operator method for finding the particular integral of simple functions. Extension of the results to higher order equations. Cauchy and Legendre equations and their solutions by reducing to equations with constant coefficients by suitable change of variable.

Module 3: Theory of Matrices

- (Review only) basic concepts about matrices. Operations involving matrices, different types of matrices. Representation of a system of linear equation in matrix form. Inverse of a matrix, Cramer's rule.

- The rows and columns of a matrix as elements of $\mathbb{R}^n$ for suitable n. Rank of a matrix as the maximum number of linearly independent rows/columns. Elementary row operations. Invariance of rank under elementary row operations. The Echelon form and its uniqueness. Finding the rank of a matrix by reducing to echelon form.

- Homogeneous and non-homogeneous system of linear equations. Results about the existence and nature of solution of a system of equations in terms of the ranks of the matrices involved.

- The eigen value problem. Method of finding the eigen values and eigen vectors of a matrix. Basic properties of eigen values and eigen vectors. Eigen values and eigen vectors of a symmetric matrix.

- Diagonalisable matrices. Advantages of diagonalisable matrices in computing matrix powers and solving system of equations. The result that a square matrix of order n is diagonalisabe (i) if and only if it has n linearly independent eigen vectors (ii) if it has n distinct eigen values. Method of diagonalising a matrix. Diagonalisation of real symmetric matrices. Similar matrices.

Text for Module 1: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
Text for Module 2: Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition, John Wiley.
Text for Module 3: David C. Lay, *Linear Algebra*, Thompson Publications, 2007.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007
4. Michael D. Greenberg, *Advanced Engineering Mathematics*, PearsonEducation, 2002.
5. George F Simmons, *Differential equations with applications and historical notes*, Tata McGraw Hill, 2003

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Geology

Semester III
Mathematics-III
(Analytic Geometry, Complex Numbers, Abstract Algebra)

CODE: MM 1331.3 — Instructional hours per week: 5
No. of Credits: 4

Module 1: Analytic Geometry

- Geometric definition of a conic-the focus, directrix and eccentricity of a conic. Classification of conics into ellise, parabola and hyperbola based on the value of eccentricity. Sketch of the graphs of conics. Reflection properties of conic sections.
  Exercise set 11.4; Questions $39 - 43$.
- Equations of the conics in standard positions. Equations of the conics which are translated from standard positions vertically or horizontally. Parametric representation of conics in standard form. Condition for a given straight line to be a tangent to a conic. Equation of the tangent and normal to a conic at a point.
- Asymptotes of a hyperbola. Equation of the asymptotes. Rectangular hyperbola and its parameric representation. Equation of tangent and normal to a rectangular hyperbola at a given point.
- Rotation of co-ordinate axes. Equation connecting the co-ordinates in the original and rotated axes. Elimination of the cross product term in a general second degree equation by suitable rotation. Identifying conics in non-standard positions represented by general second degree equation by suitable rotation of axes. The discriminant of a general second degree equationand its invariance under rotation of co-ordinate axes. The conditions on the discriminant for the general second degree equation to represent a conic, a pair of straight lines or a circle.
- Conic sections in polar coordinates. Eccentricity of an ellipse as a measure of flatness. Polar equations of conics. Sketching conics in polar coordinates. Kepler's Laws.
  Example 4 of section 11.6.

Module 2: Complex Numbers

- Review of basic results: Introduction to complex numbers, representation of complex numbers, the Argand diagram, De Moivre's theorem, evaluation of roots of complex numbers, finding n^{th} roots of unity, its properties,
- Expansion of trigonometric functions of multiples of angles, expansion of powers of trigonometric functions, separation into real and imaginary parts, Summation of series.

Module 3: Abstract algebra

- Groups–definition and examples, elementary properties, finite groups and subgroups, cyclic groups, elementary properties, symmetry of plane figures.
- Rings and fields–definition and examples,
- Vector spaces, definition and examples, elementary properties, linear dependence and independence, basis and dimension.

Text for Modules 1: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
Text for Module 2: S K Mapa, *Higher Algebra (Classical)*, Sarat Book Distributors, Kolkata.
Text for Module 3: J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications
REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. D A R Wallace, *Groups, Rings and Fields*, Springer

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Geology

Semester IV
Mathematics-IV
(Vector Analysis and Fourier Series)

CODE: MM 1431 — Instructional hours per week: 5
No. of Credits: 4

Module 1: Vector Differentiation

- (Review only) Vectors in 3-space. Addition of two vectors, multiplication of a vector by a scalar and basic properties of theseoperations. Representation in Cartesian coordinates using standard basis. Dot, cross and triple product of vectors, their significance and properties.

- Vector function of a single variable and representation in terms of standard basis. Limit of a vector function and evaluation of limit in Cartesian representation. Continuous vector functions and the idea that such functions represent oriented space curves. Examples.

- Derivative of a vector function and its geometric significance. Derivative in terms of Cartesian components. Tangent vector to a curve, smooth and piecewise smooth curves. Applications to finding the length and curvature of space curves, velocity and acceleration of motion along a curve etc.

- Scalar field and level surfaces. The gradient vector of a scalar field (Cartesian form) at a point and its geometric significance. Gradient as an operator and its properties. Directional derivative of a scalar field and its significance. Use of gradient vector in computing directional derivative.

- Vector fields and their Cartesian representation. Sketching of simple vector fields in the plane. The curl and divergence of a vector field(Cartesian form) and their physical significance. The curl and divergence as operators, their properties. Irrotational and solenoidal vector fields. Various combinations of gradient, curl and divergence operators.

Module 2: Vector Integration

- The method of computing the work done by a force field in moving a particle along a curve leading to the definition of line integral of a vector field along a smooth curve. Scalar representation of line integral. Evaluation as a definite integral. Properties. Line integral over piecewise smooth curves. Green's theorem in the plane (without proof) for a region boundedby a simple closed piecewise smooth curve.

- Oriented surfaces. The idea of flux of a vetor field over a surface in3-space. The surface integral of a vector field over a bounded oriented surface. Evaluation by reducing to a double integral. Use of cylindrical and spherical co-ordinates in computing surface integral over cylindrical and spherical surfaces.

- Stokes' theorem (without proof) for an open surface with boundary a piecwise smooth closed curve. Gauss' divergence theorem(without proof). Verification of the theorems in simple cases and their use in computing line integrals or surface integrals which are difficult to evaluate directly. Physical intrepretation of divergence and curl in terms of the velocity field of a fluid flow.

- Conservative fields and potential functions. Relation of conservative vector fields to their irrotational nature and the path- independence of line integrals in the field (without proof). Significance of these results in the case of conservative force fields such as gravitational, magnetic and electric fields. Method of finding the potential function of a conservative field.

Module 3: Fourier Series and transforms

- Periodic functions, trigonometric series, Fourier series, evaluation of Fourier coefficients for functionsdefined in $(-\infty, +\infty)$, Fourier series for odd and even functions, half range series, Fourier series for odd and even functions, Fourier series of functions defined in $(-L, +L)$.

- Fourier integrals and Fourier transforms.

Text for Modules 1 and 2: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
Text for Module 3: Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition, John Wiley. Chapter 8, Sections 1, 2, 3, 4, 8, 10.

References:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.

2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007

4. Michael D. Greenberg, *Advanced Engineering Mathematics*, PearsonEducation, 2002.

Distribution of instructional hours:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Statistics

Semester I
Mathematics-I
(Theory of equations, Infinite Series and Analytic Geometry)

CODE: MM 1131.4 Instructional hours per week: 4
No. of Credits: 3

Overview of the course:

The complementary course intended for Statistics students lays emphsis on the application of mathematical methods to Statistics. The First Module develops concepts in the theory of equations and covers the methods of solving the cubic and the quartic. The second module starts with a sequence of real numbers and goes on to discuss various tests for the convergence of an infinite series. The third Module treats analytic geometry.

Module 1: Theory of equations

- Polynomial equations and fundamental theorem of algebra (without proof). Applications of the fundamental theorem to equations having one or more complex roots, rational roots or multiple roots.
- Relations between roots and coefficients of a polynomial equation and computation of symmetric functions of roots. Finding equations whose roots are functions of the roots of a given equation. Reciprocal equation and method of finding its roots.
- Analytical methods for solving polynomial equations of order up to four-quadratic formula, Cardano's method for solving cubic equations, Ferrari's method (for quartic equations). Remarks about the insolvability of equations of degree five or more. Finding the nature of roots without solving-Des Cartes' rule of signs.

Module 2: Infinite Series

- Sequences of real numbers and limit of a sequence. Convergent and divergent sequences. Algebra of convergent sequences.Bounded and monotone sequences. The result that bounded monotone sequences are convergent (without proof). Infinite limits and limit at infinity with examples.
- Infinite series as a sequence of partial sums of a given sequence. Convergence and divergence of series. The behaviour of the series $\sum 1/n^p$. Tests of convergence–comparison test, ratio test and root test. Examples illustrating the use of these tests. Series of positive and negative terms. Absolute convergence. The result that absolute convergence impliesconvergence. Tests for absolute convergence-comparison, ratio and root tests. Alternating series and Leibnitz test for convergence.

Module 3: Analytic geometry

- (Review) Geometric definition of a conic-the focus, directrix and eccentricity of a conic. Classification of conics into ellise, parabola and hyperbola based on the value of eccentricity. Sketch of the graphs of conics.
- Equations of the conics in standard positions. Equations of the conics which are translated from standard positions vertically or horizontally. Parametric representation of conics in standard form. Condition for a given straight line to be a tangent to a conic. Equation of the tangent and normal to a conic at a point.
- Asymptotes of a hyperbola. Equation of the asymptotes. Rectangular hyperbola and its parameric representation. Equation of tangent and normal to a rectangular hyperbola at a given point.
- Rotation of co-ordinate axes. Equation connecting the co-ordinates in the original and rotated axes. Elimination of the cross product term in a general second degree equation by suitable rotation. Identifying conics in non-standard positions represented by general second degree equation by suitable rotation of axes. The discriminant of a general second degree equation and its invariance under rotation of co-ordinate axes. The conditions on the discriminant for the general second degree equation to represent a conic, a pair of straightlines or a circle.

Texts for Modules 1 and 2:

1. Barnard and Child, *Higher Algebra*, Macmillan.
2. S K Mapa, *Higher Algebra (Classical)*, Sarat Book Distributors, Kolkata.

Texts for Module 3:
Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Statistics

Semester II
Mathematics-II
(Differential Calculus)

CODE: MM 1231.4 | Instructional hours per week: 4
No. of Credits: 3

Overview of the course:
The complementary course intended for Statistics students lays emphsis on the application of mathematical methods to Statistics. The two modules on Differential Calculus links the topic to *the real world and the student's own experience* as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. Applications to Statistics on the lines of those in Physics as given in the text could be obtained from the net. The third Module on Integral Calculus reviews basic integration techniques and covers several applications of integration. I t also treats multiple integrals. The emphasis should be on applications to statistical problems.

Module 1: Differentiation with applications to Statistics-I

- Functions and graphs of functions with examples from Statistics. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid.
  Exercise set 1.8; Questions $31 - 34$.

- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits. Indeterminate forms of the type $0/0$.
  Exercise set 2.1; Questions 27 and 28.

- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.
  Exercise set 3.1; Questions 1, 2 and 16.

- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates.Local linear approximation. Differentials.
  Examples $1 - 6$.Exercise set 3.8; Questions $53 - 55$.

- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.
  Examples $1 - 7$. Exercise set 4.4; Questions 8, 9, $30 - 32$.

- Absolute maxima and minima. Applied maximum anmd minimimum problems.
  Exercise set 4.6; Questions 47 and 48.

- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems 4.1.2 and 4.83 (consequences of the Mean Value Theorem).

- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function. Graphs and applications involving logarithmic and exponential functions.
  Exercise set 7.4; Question 50.

- L'Hospital's Rule for finding the limits (without proof) of indeterminate forms of the type $0/0$ and ∞/∞. Analysing the growth of exponential functions using L'Hospital's Rule. Indeterminate forms of type$0 \cdot \infty$ and $\infty - \infty$ and their evaluation by converting them to $0/0$ or ∞/∞ types. Indeterminate forms of type 0^0, ∞^0 and 1^∞.

- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.

Module 2: Differentiation with applications to Statistics-II

Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximatingfunctions by polynomials. Taylor series and Maclaurin's series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.

Module 3: Differentiation with applications to Statistics-III

- Functions of two variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians.
  Exercise set 14.3; Questions 47 and 48.Exercise set 14.4; Question 50.Exercise set 14.5; Question 42.

- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.
  Exercise set 14.9; Question 20.

TEXT: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.

2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

3. Peter V. O' Neil, *Advanced Engineering Mathematics*, ThompsonPublications, 2007

4. Michael D. Greenberg, *Advanced Engineering Mathematics*, PearsonEducation, 2002.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Statistics

Semester III
Mathematics-III
(Integration and Complex Numbers)

CODE: MM 1331.4

Instructional hours per week: 5
No. of Credits: 4

Module 1: Integration with applications to Statistics-I

- Indefinite integrals (Review only), integral curves, integration from the view point of differential equations, direction fields
  Exercise set 5.2; Questions 43 and 44
- (Review only) Definite integral and the Fundamental Theorem of Calculus.
- Techniques of integration.
- Rectilinear motion: finding position and velocity by integration. Uniformly accelerated motion. The free-fall model. integrating rates of change. Displacement in rectilinear motion. Distance travelled in rectilinear motion. Analysing the velocity versus time curve. Average value of a continuous function. Average velocity revisited.Exercise set 5.7; Questions 3, 4, 5, 6, 29 and 55
- Use of definite integrals in finding area under curves, area between two curves, volume of revolution, arc length and surface area of a solid of revolution.

Module 2: Integration with applications to Statistics-II

- The idea of approximating the volume under a bounded surface in 3-space by volumes of boxes, leading to the definition of double integrals of functions of two variables over bounded regions. Evaluation of double integrals by iterated integrals. Evaluation by changing to polar co-ordinates and by suitably changing order of integration in the iterated integral. Applications to finding the volume of solids under bounded surfaces.
- Triple integrals over bounded regions in 3-space. Evaluation by iterated integrals. Cylindrical coordinates and spherical coordinates and their relation to Cartesian coordinates. Use of cylindrical and spherical co-ordinates in evaluating triple integrals. Applications of triple integrals to finding volumes of solid objects.

Module 3: Complex Numbers

- Review of basic results: Introduction to complex numbers, representation of complex numbers, the Argand diagram, De Moivre's theorem, evaluation of roots of complex numbers, finding n^{th} roots of unity, its properties,
- Expansion of trigonometric functions of multiples of angles, expansion of powers of trigonometric functions, separation into real and imaginary parts, Summation of series.

Text for Modules 1 and 2 : Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley
Text for Module 3: S K Mapa, *Higher Algebra (Classical)*, Sarat Book Distributors, Kolkata.

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 35 hours; Module 2: 35 hours; Module 3: 20 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Statistics

Semester IV
Mathematics-IV
(Linear Algebra)

CODE: MM 1431.4 | Instructional hours per week: 5
No. of Credits: 4

Module 1: Vector Spaces over $\mathbb{R}$

- Vector in 3-space as an ordered triple of real numbers. Addition of two vectors and multiplication of a vector by a scalar. Algebra of vectors involving addition and scalar multiplication. The norm of a vector. The dotproduct and orthogonal vectors. Geometric interpretation of these concepts andtheir connection to the traditional method of representing a vector in terms ofstandard unit vectors.

- The $n-$tuple as a generalisation of ordered triple and the space $\mathbb{R}^n$of all $n-$tuples. Addition of two $n-$tuples and multiplication ofan $n-$tuple by a scalar. Listing of the algebraic properties of $\mathbb{R}^n$ thatmakes it a vector space. Dot product of $n-$tuples and orthogonality. TheCauchy-Schwarz inequality in $\mathbb{R}^n$.

- Sub space of $\mathbb{R}^n$. Geometric meaning of subspaces in $\mathbb{R}^2$.and $\mathbb{R}^3$. Linear dependence and independence of vectors in $\mathbb{R}^n$. Basis anddimension and the standard basis of $\mathbb{R}^n$. Orthogonal and orthonormal bases.Representation of an arbitrary vector in an orthonormal basis. The Gram-Schmidtorthogonalisation process.

Module 2: Theory of Matrices

- (Review only) basic concepts about matrices. Operations involving matrices,different types of matrices. Representation of a system of linear equation inmatrix form. Inverse of a matrix, Cramer's rule.

- The rows and columns of a matrix as elements of $\mathbb{R}^n$ for suitable n. Rank of a matrix as the maximum number of linearly independent rows/columns. Elementary row operations. Invariance of rank under elementary row operations. The Echelon form and its uniqueness. Finding the rank of a matrix by reducing to echelon form.

- Homogeneous and non-homogeneous system of linear equations. Resultsabout the existence and nature of solution of a system of equations in terms ofthe ranks of the matrices involved.

- The eigen value problem. Method of finding the eigen values and eigenvectors of a matrix. Basic properties of eigen values and eigen vectors. Eigen values and eigen vectors of a symmetric matrix. The result that theeigen vectors of a real symmetric matrix form an orthogonal basis of $\mathbb{R}^n$.

- Diagonalisable matrices. Advantages of diagonalisable matrices incomputing matrix powers and solving system of equations. The result that asquare matrix of order n is diagonalisabe (i) if and only if it has $n$linearly independent eigen vectors (ii) if it has n distinct eigen values. Method of diagonalising a matrix. Diagonalisation of real symmetric matrices.

- Quadratic forms in $\mathbb{R}^n$ and matrix of quadratic forms. Canonical formof a quadratic form and the principal axes theorem. Geometric meaning ofprinciple axes theorem for quadratic forms in $\mathbb{R}^2$. Use of these results inidentifying the type of a conic that a general second degree equation mayrepresent.

Module 3: Linear Transformations

- Linear transformations from $\mathbb{R}^n$ into $\mathbb{R}^m$. Matrix of alinear transformation relative to a given pair of bases and lineartransformation defined by a matrix. Characterisation of linear transformationsfrom $\mathbb{R}^n$ into $\mathbb{R}^m$.

- Linear transformations from $\mathbb{R}^n$ into $\mathbb{R}^n$ and matix of suchtranformations. Matrix representation of simple tranformations such asrotation, reflection, projection etc. on the plane. Relation between matricesof a given transformation relative to two different bases. Method of choosing a suitable basis in which the matrix of a given transformation has the particularly simple form of a diagonal matrix.

Text: David C. Lay, *Linear Algebra*, Thompson Publications, 2007References:

1. T S Blyth and E F Robertson: *Linear Algebra*, Springer, Second Ed.

2. Peter V. O' Neil: *Advanced Engineering Mathematics*, ThompsonPublications, 2007

3. Michael D. Greenberg: *Advanced Engineering Mathematics*, PearsonEducation, 2002.

Distribution of instructional hours:

Module 1: 30 hours; Module 2: 30 hours; Module 3: 30 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Economics

Semester I
Mathematics for Economics-I

CODE: MM 1131.5 | Instructional hours per week: 3
No. of Credits: 2

Overview of the course:
The complementary course intended for Economics students lays emphsis on the increased use of mathematical methods in Economics. The first Module of the first semester course discusses the basic concepts of functions, limits and continuity, which is essential to understand what is to follow in subsequent Modules. The second Module is on Differentiation. Applications to Economics abound in this area. The concepts should therefore be carefully motivated with suitable examples.

Module 1: Functions, Limits and Continuity

- Functions: Definition and examples of functions, domain and range of a function, graph of a function, notion of implicit and explicit functions, demand functions and curves, total revenue functions and curves, cost functions and curves, indifference function, indifference curves for flow of income over time.
- Limits and continuity of functions:Notion of the limit of a function with sufficient examples, algebra of limits (No proof), theorems on limits :$\lim_{x\to a}\dfrac{x^n-a^n}{x-a}=nx^{n-1}$, $\lim_{x\to 0}\dfrac{\sin x}{x}=1$,$\lim_{x\to 0}\dfrac{e^x-1}{x}=1$, $\lim_{x\to 0}\dfrac{a^x-1}{x}=\log a$, for $a>0$(No proof), definition and examples of continuous functions, discontinuity, examples, geometrical meaning of continuity

Module 2: Differentiation-I

- Differentiation: Differentiation of functions of one variable, derivative as a rate measure, rules of differentiation, derivative of a function at a point, product rule, quotient rule, function of a function rule, derivatives of standard functions, derivatives and approximate values, geometrical interpretation of the derivative, applications in economics (such as marginal revenue, marginal cost),

REFERENCES:

1. R G D Allen, *Mathematical Analysis for Economics*, AITBS Publishers, D-2/15. Krishnan Nagar, New Delhi

2. Taro Yamane, *Mathematics for Economists, An Elementary Survey*, PHI, New Delhi.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Economics

Semester II
Mathematics for Economics-II

CODE: MM 1231.5 — Instructional hours per week: 3

No. of Credits: 3

Overview of the course:
The first module on differentiation discusses differntials, increasing and decreasing functions and maxima and minima, along with several applications. The second module is on partial differentiation. It considers the maxima and minima of functions of two varibles and these are readily applied to problems in Economics.

Module 1: Differentiation-II

- Further differentiation:Successive derivatives of elementary functions, differentials and approximations, increasing and decreasing functions, turning points, points of inflexion, convexity of curves, maxima and minima of functions of one variable, the problem of average and marginal values,problems of monopoly and duopoly in economic theory.

Module 2: Partial Differentiation

- Partial Differentiation: Functions of several variables, Definition and examples partial differentiation of functions of two variables, maxima and minima of functions of many variables, Lagrangian multiplier method of maxima and minima of functions, illustrations from economics, geometrical interpretation of partial derivatives, total differentials, derivatives of implicit functions, higher order partial derivatives, homogeneous functions, applications(maxima and minima problems) in economics,

REFERENCES:

1. R G D Allen, *Mathematical Analysis for Economics*, AITBS Publishers, D-2/15. Krishnan Nagar, New Delhi

2. Taro Yamane, *Mathematics for Economists, An Elementary Survey*, PHI, New Delhi.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Economics

Semester III
Mathematics for Economics-III

CODE: MM 1331.5 Instructional hours per week: 3
No. of Credits: 3

Overview of the course:

The course follows the trends set in the first two semester. Integration techniques, definite integrals and approximate integration are discussed in the first module, highlighting applications to Economics. Various infinite series form the content of the second module.

Module 1: Integration

- Integration : Integral as an antiderivative, integration by substitution, integration by parts, definition of the definite integral, definite integrals and approximate integration (Simpson's rule and trapezoidal rule),total cost, marginal cost, capitalisation of an income flow, law of growth, Domar's models on public debt and national income.

Module 2: Series

- Series: geometric, binomial, exponential and logarithmic series, Taylor's formula, Taylor series, extension to many variables.

REFERENCES:

1. R G D Allen, *Mathematical Analysis for Economics*, AITBS Publishers, D-2/15. Krishnan Nagar, New Delhi

2. Taro Yamane, *Mathematics for Economists, An Elementary Survey*, PHI, New Delhi.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Economics

Semester IV
Mathematics for Economics-IV

CODE: MM 1431.5 — Instructional hours per week: 3
No. of Credits: 3

Overview of the course

The two modules in this course treat differential equations, the solutions of which are important in most mathematical models. First order differential equations are considered in the first module, whereas second order differenetial equations with constant coefficients, together with the Euler equation are dealt with in the second module.

Module 1: Differential Equations-I

- Differential Equations: Formulation of differential equations, geometrical interpretation of a differential equation representing a family of curves, First order equations, Linear equations, Variables separable, Homogeneous equations.

Module 2: Differential Equations-II

- Differential equations of higher order: Second order differential equations with constant coefficients with RHS as one of $x, e^{ax}, \sin ax, \cos ax$, Euler equations, applications in economics, Domar's capital expansion model, equilibrium of a market and stability of equilibriumof a dynamic market.

REFERENCES:

1. R G D Allen, *Mathematical Analysis for Economics*, AITBS Publishers, D-2/15. Krishnan Nagar, New Delhi

2. Taro Yamane, *Mathematics for Economists, An Elementary Survey*, PHI, New Delhi.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 27 hours; Module 2: 27 hours